

Teaching Basic Vector Concepts: A Worksheet for the Recovery of Students' Vector Understanding

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Abstract

The investigation of high-school students' qualitative understanding of graphical vector addition in one and two dimensions has revealed that students did not grasp the important basic concepts about vectors. Based on their misunderstanding, we had designed a worksheet in order to help them learn such topics in class more effectively. The content in the worksheet includes vector directions, magnitude, addition and subtraction. A 2-page worksheet was constructed and distributed to 3 classes of high-school students in a large public school (n=103). Students took approximately 30-40 minutes to complete the worksheet. A seven-item diagnostic quiz was applied as a pre/post-test in order to evaluate the effectiveness of the worksheet. Using paired-sample t-test, results from the diagnostic quiz showed that the students' average post-test scores for all classes were significantly higher than their average pre-test scores ($p < 0.000$). In addition, the class average normalized gains ranged from 0.69-0.76 which was considered as high gains that might be rarely reached by conducting classes even with active learning activities.

Keywords: Vector, Vector addition; Vector subtraction; Resultant vectors; Worksheet

Introduction

Some previous studies have shown that student difficulty in learning physics is mainly caused by a lack of skills and understanding about mathematics, especially vectors (Flores-García, Alfaro-Avena, & Dena-Ornelas, 2008). Students require a good grasp of basic vector concepts to succeed in a physics course (Sheets, 1998). While understanding vector is necessary, physics education researchers also found that students still hold misconception about vectors although they have studied it before (Knight, 1995; Nguyen & Meltzer, 2003; Flores, Kanim, & Kautz, 2004; Shaffer & McDermott, 2005; Heckler & Scaife, 2015). Lacking of understanding about vectors might cause many serious problems when the vector concepts are embedded in almost all physics concepts (Aguirre, 1988; Aguirre & Fankin, 1989; Zavala & Barniol, 2013; Barniol & Zavala, 2015). For example, student might be unable to use vectors in solving problems about kinematic quantities, forces and fields. Not only the introductory students but also many precollege teachers and even graduate students, vector operation difficulties were observed. With or without physics context, most students have difficulties with vectors (Barniol & Zavala, 2010; Barniol & Zavala, 2012; Barniol, Zavala, & Hinojosa, 2013; Barniol & Zavala, 2014). Thus, recovering students' understanding

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of vector concept in the beginning of a physics course might reduce their difficulties in learning physics topics.

In Thailand, students also still hold misconception after instruction, especially in graphical vector addition for both one and two dimensions. The finding was presented by our work in 2011 (Wutchana & Emarat, 2011). Using the diagnostic quiz previously used by Nguyen and Meltzer (2003) and deeply analyzing student responses from two free-response problems of graphical vector addition in one and two dimensions, we found that there was a variety of student misunderstanding about vector properties and operation. Most students did not grasp the important concept about the vector direction and how a vector may be moved while its magnitude and direction are preserved. Some students ignored the direction of resultant vectors. Many students were confused about the “tip-to-tail” strategy. Most of them thought that adding two vectors meant just attaching them to each other without knowing how to do it properly. The most crucial things we found was that some students made an attachment between two added vectors correctly but finally they gave the wrong direction of a resultant vector. The findings are consistent with the work of Knight (1995) and also the work of Nguyen and Meltzer (2003).

There were four important points suggested from our previous study (Wutchana & Emarat, 2011). Before teaching physics, instructors should make sure that their students: 1) realize that vector quantities comprise both magnitude and direction; 2) understand that a vector can be moved and while moving a vector, its magnitude and direction must be preserved; 3) know that when adding two vectors, one vector’s tip must be attached to the other vector’s tail and; 4) are clear that the direction of the resultant vector points from the tail of the initial vector to the tip of the second vector. According to these four points, we had designed a 2-page worksheet based on the students’ misunderstanding to resolve the problems and recover their understanding of basic vector concepts. It was expected that this could help reduce future difficulties they might encounter and could help them learn physics more effectively.

Methodology

A pre-experimental research design was conducted to see the effect of the worksheet to students’ understanding of basic vector concepts. In this study, we focused on the worksheet instruction as an experimental treatment and students’ responses to a diagnostic quiz as a dependent variable. The difference between students’ pre- and post-test scores would be used to reflect the effectiveness of the worksheet. In equivalent way, we collected data from 3 classes of high-school students. Details of the *worksheet design; diagnostic quiz; data collection; and data analysis*; are presented in the followings.

Worksheet design

A worksheet was designed following the four points suggested from our previous study (Wutchana & Emarat, 2011) in order to recover students’ basic vector concepts and make sure that the students are able to carry out vector addition and subtraction graphically in one and two dimensions before starting their physics courses. There were three parts in the worksheet: 1) vector quantities; 2) vector addition in one and two dimensions and; 3) vector subtraction in one and two dimensions. The designed worksheet can be found in the appendix.

Part 1: Vector quantities

To make students realize that vector quantities comprise magnitude and direction, students were assigned to draw vectors that contained the same magnitude with the given one as well as the vectors with the same direction as the one shown in the problem. This is

expected to make them aware that vector quantities should be composed of both magnitude and direction. In addition, to make them understand that a vector can be moved, an exercise on moving vectors to any point was successively given before they learned how to add vectors. The students were assured that vectors can be moved and they must also ensure that the magnitude and direction of the vector are preserved while being moved.

Part 2: Vector addition in one and two dimensions

This part is to ensure that students know that when adding two vectors, one vector's tip must be attached to the other vector's tail and the direction of the resultant vector points from the tail of the initial vector to the tip of the second vector. The major problem found was that students did not know the correct way to attach the two vectors. They simply attached the vector tails together and then made the connection between the vector tips to form a resultant vector (Wutchana & Emarat, 2011). Many students added the vectors by creating a wrong triangle. Most of them thought that adding two vectors meant just attaching them to each other without knowing how to do it properly. To clarify their understanding that adding vectors is not just connecting those vectors, we made sure in the worksheet that all students know which part is the tip or tail of the given vector and which vector is the initial one or the second one. This is to prepare and remind them before connecting the vectors graphically using the "tip-to-tail" strategy. After this, all students were asked to add given vectors in one and two dimensions starting with adding two and three vectors in one dimension, and then two and three vectors in two dimensions.

To add any two vectors graphically, students have to start with drawing an initial vector by preserving its magnitude and direction. Then, connect the second vector's tail to the initial vector's tip and while doing this the magnitude and direction of the second vector should also be preserved. Finally, a new vector is formed by drawing an arrow starting from the initial vector's tail to the second vector's tip. This new created vector is called a resultant vector.

Right here, an optional teaching method could be applied. A rubber band could be used to find a resultant vector of graphical vector addition (Wutchana & Emarat, 2014). After the students graphically made attachment between two or more added vectors, they could find the resultant vector by stretching the rubber band following the connected vectors. This means holding one end of the rubber band to stay still at the first vector tail. Then stretch the other end attached with the arrowhead paper following each of the connected vector tips until it reaches the final vector tip. The stretching length of the rubber band represents the magnitude of the resultant vector and the arrowhead on the rubber-band end points in the direction of the resultant vector. The students could draw the resultant vector in the worksheet following the rubber band appearance. They will see immediately that the direction of the resultant vector will point from the started vector tail to the final vector tip. Although a rubber band method is an option, it is suggested that the instructor adopt this into their teaching along with the worksheet. We found that this helps promote active learning in class. More detail about how to create a rubber band vector, as well as how to use it, can be found in Wutchana and Emarat (2014).

Part 3: Vector subtraction in one and two dimensions

This part is to ensure that students realize that an opposite direction vector is needed before starting the vector subtraction. Students are asked to draw a vector with the same magnitude but opposite direction as the given one and this will be added to the other vector.

To subtract any two vectors graphically, students have to start with drawing an initial vector whose magnitude and direction is preserved. Then, for the second vector that will be subtracted from the initial one, draw it with the same magnitude but with the opposite

direction. Then, connect this second vector's tail to the initial vector's tip. Finally, a resultant vector is formed by drawing an arrow starting from the initial vector's tail to the second vector's tip. Students were shown that the subtraction algorithm is just adding the initial vector with the second vector whose magnitude remains the same but its direction is now opposite.

A rubber band could also be used to find a resultant vector of graphical vector subtraction. In this part of the worksheet, students were asked to subtract all given vectors in one and two dimensions.

Diagnostic quiz

A diagnostic quiz used in this study had been translated from the work of Nguyen and Meltzer (2003) to measure the students' understanding of vector direction, magnitude, addition and subtraction presented in graphical forms. The quiz includes problem #1: Vector magnitude, problem #2: Vector direction, problem #3: Qualitative vector addition, problem #4: One-dimensional vector addition, problem #5: Two-dimensional vector addition, problem #6: Two-dimensional vector subtraction, and problem #7: Comparison of resultant magnitude. Students were asked to give a free response or to select multiple options from a list. Except for problems #3 and #7, the students were given possible choices.

Data collection

Basically, the vector concepts are included in the introduction chapter of a physics textbook. Both at the high-school and university levels, students have to study a lesson of vectors before starting to learn physics. In this study, data was separately collected from 3 classes (C1, C2, and C3) of grade ten students (n=34, n=32, and n=37) from a large public high school in Thailand. The study was operated during the school's preparation camp before starting the first semester of the 2014 academic year. The school camp is normally conducted in the middle of May for all freshmen in every academic year. All students have previously completed the basic vector concepts as they are embedded in a course of science and also in a mathematics lesson in middle-school level.

During the first meeting hour of the camp, the diagnostic quiz translated from the work of Nguyen and Meltzer (2003) was administered to all students in each class. The students spent 15 minutes to complete the quiz. The designed worksheet was then distributed to the students individually. Taking approximately 30-40 minutes, the students were recovered their vector understanding following the worksheet step accompanied with the teacher guidance. After completing the worksheet, the diagnostic quiz again was administered to the students as a post-test in order to evaluate the effectiveness of the worksheet instruction. Like the pre-test, the students spent 15 minutes to complete the test.

Data analysis

The students' pre- and post-test scores were compared using paired-sample t-test to see whether the differences between them are statistically significant or not. Students' average learning gains were also calculated to see the classes' progression. The student's learning gain was assessed by comparing the difference between their pre- and post-test scores to the maximum possible gain and this is called normalized gain (Hake, 1998). Normalized gain (g) is defined as the change in score divided by the maximum possible increase as presented in following.

$$g = \frac{\text{posttest_score} - \text{pretest_score}}{\text{maximum_score} - \text{pretest_score}}$$

In this study, the class average normalized gains ($\langle g \rangle$) were calculated from the average of students' normalized gains (Bao, 2006). They were used to represent the effectiveness of the worksheet instruction. However, for the students who had full scores for both pre-test and post-test, their scores would be removed from the data set because the student performance was beyond the scope of the measurement instrument (Marx & Cummings, 2006). To see the progression in each problem, the normalized gains were then calculated separately for each problem by comparing the differences of students' correct responses between pre- and post-test to the maximum possible gain of that problem.

The class average normalized gains were divided into three regions; high gain is where $\langle g \rangle$ is greater than 0.7 ($\langle g \rangle_{\text{high}} > 0.7$), medium gain with $\langle g \rangle$ between 0.3 and 0.7 ($0.3 \leq \langle g \rangle_{\text{medium}} < 0.7$) and low gain with $\langle g \rangle$ less than 0.3 ($\langle g \rangle_{\text{low}} < 0.3$). After Hake had proposed this analyzing method in 1998, normalized gain has been widely used by many Physics Education researchers (Celletta & Phillips, 2005; Stewart & Stewart, 2010; Celleta, Phillipss, Jeffery & Jeff, 2011).

Results and Discussion

The pre-test could establish a baseline for students' understanding before the instruction and post-test could represent students' understanding after the instruction. Differences between pre- and post-test could indicate the effectiveness of the worksheet instruction.

Table 1 shows each class's pre- and post-test average scores on the seven-item quiz. The student numbers in each class were 34, 32, and 37, respectively. The difference between pre- and post-test average scores are statistically significant using paired-sample t-test ($p < 0.000$). It could be said that the students' understanding were improved or their understanding of basic vector concepts were recovered after the worksheet instruction. To see the classes' progression, the class average normalized gains were calculated.

For the students who had maximum scores for both pre- and post-test, their scores should be removed from the data set. Therefore, student numbers in each class were changed to be 32, 22, and 37, respectively. As presented in table 1, the class average normalized gains, $\langle g \rangle$, are 0.70, 0.76, and 0.69 which could be considered as high gain ($0.3 \leq \langle g \rangle_{\text{medium}} < 0.7$ and $\langle g \rangle_{\text{high}} > 0.7$) (Hake, 1998) that could rarely be reached.

Table 1. Class average pre- and post-test scores on the diagnostic quiz (maximum score is 7)

| Classes | Pre-test | Post-test | Sig | $\langle g \rangle$ |
|---------|----------|-----------|--------|---------------------|
| C1 | 4.66 | 6.34 | 0.000* | 0.70 |
| C2 | 4.31 | 6.59 | 0.000* | 0.76 |
| C3 | 4.65 | 6.16 | 0.000* | 0.69 |

* Significant 2-tailed

To see the progression in each problem, the normalized gains were calculated separately for each problem as presented in table 2.

Not only that the pre-test establishes a baseline for student understanding before the instruction but it also provides insights into the nature of specific difficulties. Although the students have studied basic vector concepts before, numbers of their correct responses to the pre-test were not quite high. Considering their pre-correct responses, low scores were found in problems #2 and #6 which are vector direction and two-dimensional vector subtraction.

Table 2. The class normalized gains and number of the students' correct responses to each problem of the diagnostic quiz during pre- and post-test

| Class Problem | C1 (n=32) | | | C2 (n=22) | | | C3 (n=37) | | |
|------------------|-----------|------|------|-----------|------|------|-----------|------|------|
| | Pre | Post | <g> | Pre | Post | <g> | Pre | Post | <g> |
| #1 | 25 | 31 | 0.46 | 16 | 22 | 1.00 | 25 | 30 | 0.42 |
| #2 | 14 | 20 | 0.25 | 11 | 14 | 0.27 | 16 | 22 | 0.29 |
| #3 | 27 | 29 | 0.18 | 20 | 22 | 1.00 | 31 | 34 | 0.50 |
| #4 | 29 | 31 | 0.22 | 19 | 22 | 1.00 | 31 | 33 | 0.33 |
| #5 | 20 | 32 | 0.67 | 11 | 22 | 1.00 | 22 | 31 | 0.60 |
| #6 | 17 | 30 | 0.62 | 8 | 21 | 0.93 | 12 | 31 | 0.76 |
| #7 | 17 | 30 | 0.62 | 10 | 22 | 1.00 | 20 | 32 | 0.71 |

Problem #2 asks students to select vectors that have the same direction with the given one. Numbers of the students providing correct responses were low for the pre-test. It indicates that previous courses could not clarify the students' understanding of this particular topic. Deep analysis of the students' responses indicates further that common student error is selecting a vector with its direction just closing to the given vector but not exactly the same. Many physics instructors thought that this concept is not difficult and so they paid less attention and went over this point quickly causing some difficulties to students. However, what we found here shows that the student did not realize that vectors of the same direction have the same angle with respect to the vertical or the horizontal axis. This is consistent with the finding of Barniol and Zavala (2009). This revealed that the students might not be aware of these properties of vectors. After the worksheet instruction, numbers of students' correct responses to this problem were increased a little for every class. The class normalized gains were closed to the medium gain region (0.25, 0.27, and 0.29) (Hake, 1998). This result suggests that there are still rooms to be improved about this basic concept of vector.

Problem #6 is about vector subtraction in two dimensions. Low number of the students' correct responses to this problem represented that the students came to classes with low understanding about subtracting vectors whereas high normalized gains turned to appear after the worksheet instruction. This means that some students' common errors on the vector subtraction might be recovered by the worksheet.

Considering the numbers of the students' correct responses to each problem, low gains were also found in problems #3, and #4 for the students in group C1. Possible reasoning could be that the numbers of the students' correct responses to the pre-test problems were already quite high and therefore it was more difficult to improve their understanding to be even better.

According to the above presented results, numbers of the students' correct responses were increased for every class and every problem. It could be noted that the worksheet instruction could help recover students' understanding of vector magnitude (problem #1). More students could identify the vectors that contained equal magnitude even when their directions differed. The normalized gain for this problem was considered to be a medium gain. Especially for graphical vector addition and subtraction in two dimensions (problems #5 and #6), the students learned these concepts very well. Their normalized gains for these two problems were ranged from 0.60 to 1.00. These were considered as medium and high gains ($\langle g \rangle_{\text{high}} \geq 0.7$). The worksheet instruction was also good for recovering understanding of the resultant vector magnitude (problem #7) since a number of students' correct response reached 80% after the instruction. The normalized gains were also ranging from a medium to high region; from 0.62 to 1.00.

Conclusions

The most crucial thing gained from this study is that spending only 30-40 minutes with the 2-page worksheet could help recover students' understanding of basic vector concepts quite effectively. Conducting classes with the worksheet, students could grasp all important vector concepts. Students realize that vector quantities comprise both magnitude and direction. They understand that a vector can be moved and while moving a vector, its magnitude and direction must be preserved. They also know that when adding two vectors, one vector's tail must be attached to the other vector's tip. The students are clear that the direction of the resultant vector points from the tail of the initial vector to the tip of the final vector. The students realize about opposite direction vectors and understand that subtraction algorithm is just adding the initial vector with the second vector whose direction is now opposite but the magnitude stays the same. This might reduce some serious problems about basic vector operation when learning kinematic quantities, forces, fields and so on. It can be confirmed by the students' post-test scores on the diagnostic quiz which are significantly higher than their pre-test scores. The post-test correct responses are higher than the pre-test in every problem as well. After the worksheet instruction, the normalized gains were in the medium and high gains, except for the concept of vector direction. This suggests that further improvement of the worksheet in the part of vector quantities could be conducted.

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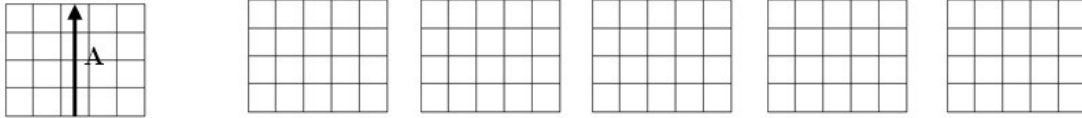
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Appendix: Worksheet

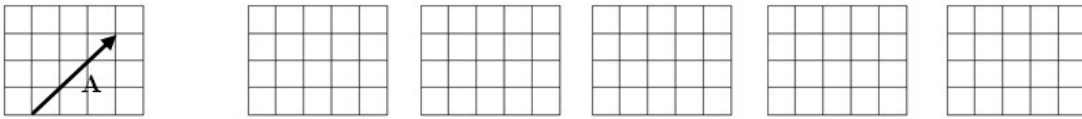
Graphical vector addition and subtraction

Part 1: Vector quantities

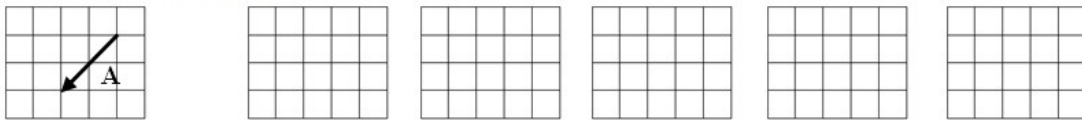
Draw the vectors that have the same **magnitude** as \vec{A}



Draw the vectors that have the same **direction** as \vec{A}

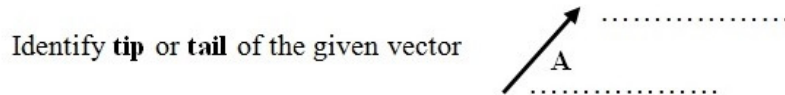


Move \vec{A} to any different position (a vector can be moved and while moving a vector, its magnitude and direction must be preserved)



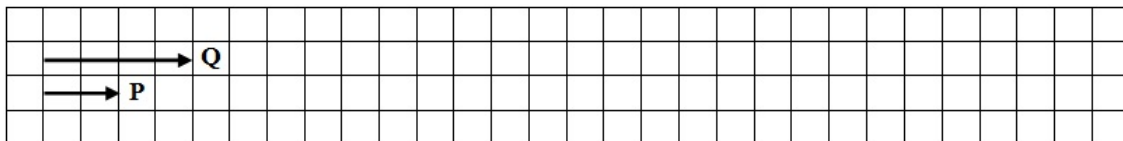
Part 2: Vector addition in one and two dimensions

To add any two vectors graphically, students have to start with drawing an initial vector while its magnitude and direction is preserved. Then, connect the second vector's tail to the initial vector's tip and while doing this the magnitude and direction of the second vector should also be preserved. Finally, a new vector is formed by drawing an arrow from the initial vector's tail to the second vector's tip. This new created vector is called a resultant vector.

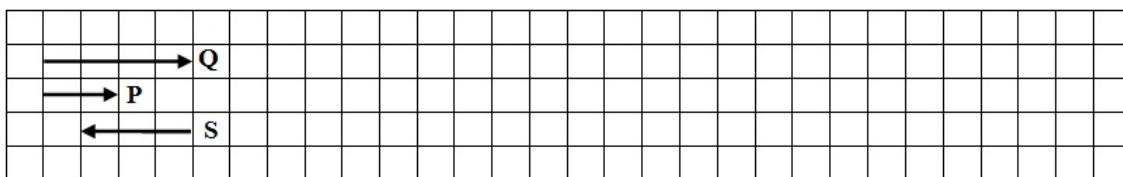


Draw \vec{R} where $\vec{R} = \vec{P} + \vec{Q}$ and identify which (\vec{P} or \vec{Q}) is the initial vector or second vector :

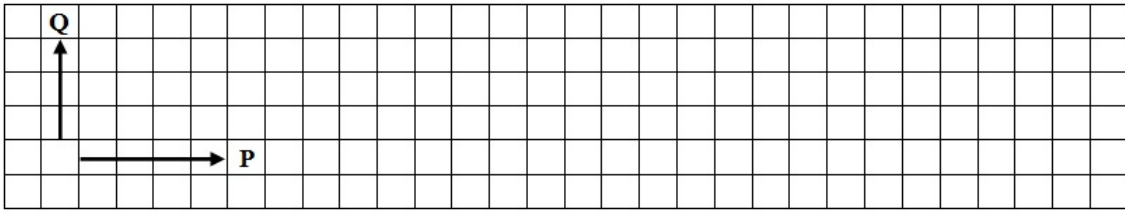
\vec{P} is the vector, \vec{Q} is the vector



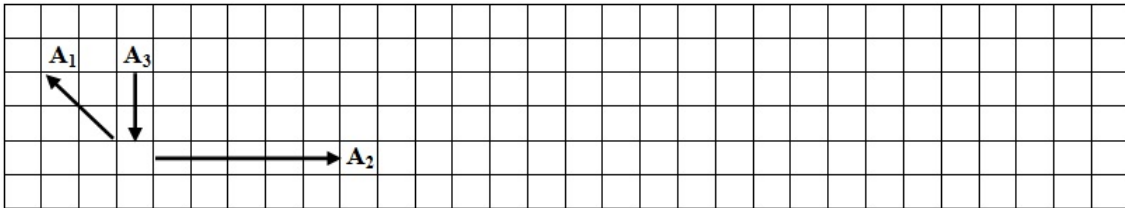
Draw \vec{R} where $\vec{R} = \vec{P} + \vec{Q} + \vec{S}$



Draw \vec{R} where $\vec{R} = \vec{P} + \vec{Q}$

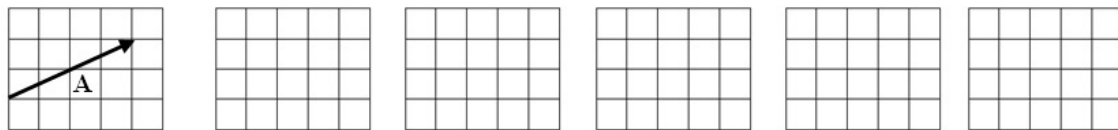


Draw \vec{R} where $\vec{R} = \vec{A}_1 + \vec{A}_2 + \vec{A}_3$



Part 3: Vector subtraction in one and two dimensions

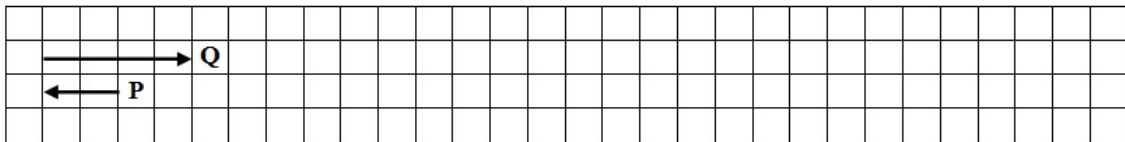
Draw the vectors that have the same magnitude as \vec{A} but pointing in the opposite direction



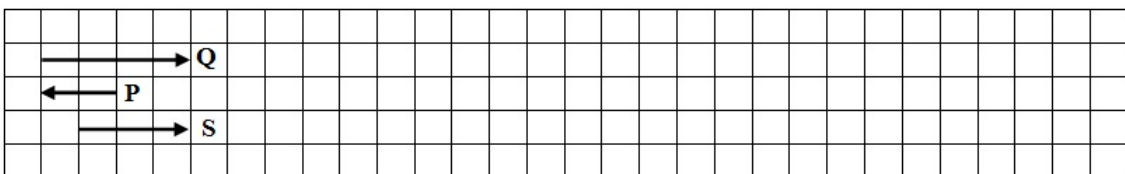
To subtract any two vectors graphically, students have to start with drawing an initial vector whose magnitude and direction is preserved. Then, draw the second vector with the opposite direction but its magnitude remains the same. Connect this second vector's tail to the initial vector's tip. Finally, the resultant vector is formed by drawing an arrow from the initial vector's tail to the second vector's tip.

Be cleared that the subtraction algorithm is just adding the initial vector with the second vector whose direction is now opposite but its magnitude stays the same.

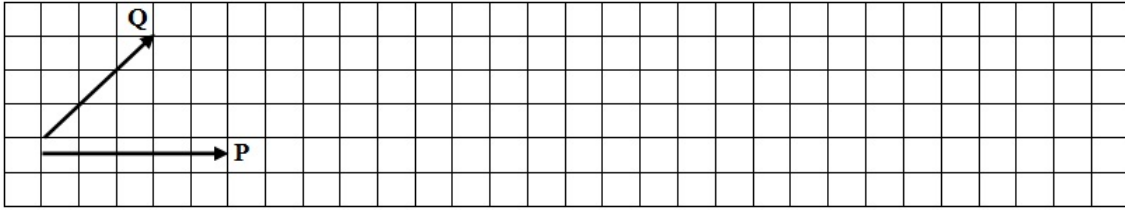
Draw \vec{R} where $\vec{R} = \vec{P} - \vec{Q}$ and identify which (\vec{P} or \vec{Q}) is the initial vector or second vector :
 \vec{P} is the vector, \vec{Q} is the vector



Draw \vec{R} where $\vec{R} = \vec{P} + \vec{Q} - \vec{S}$



Draw \vec{R} where $\vec{R} = \vec{P} - \vec{Q}$



Draw \vec{R} where $\vec{R} = \vec{B}_1 + \vec{B}_2 - \vec{B}_3$

