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Physics and Mathematics models in a co-disciplinary Study and Research Paths (SRPs) in the pre-service teacher education

Viviana Carolina Llanos*
Universidad Nacional del Centro de la Provincia de Buenos Aires (UNICEN), Argentina
Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina
vcllanos@exa.unicen.edu.ar

María Rita Otero
Universidad Nacional del Centro de la Provincia de Buenos Aires (UNICEN), Argentina
Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina
rotero@exa.unicen.edu.ar

María Paz Gazzola
Universidad Nacional del Centro de la Provincia de Buenos Aires (UNICEN), Argentina
Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina
mpgazzola@exa.unicen.edu.ar

Abstract
We present results of an implementation of Study and Research Path (SRP) carried out into a pre- service mathematics teacher-training course at University level. A co-disciplinary SRP from the generating question Q_0: Why did the Movediza stone in Tandil fall? Requires developing both physical and mathematical models to build a possible answer to this question. Some conclusions concerning on the restrictions and relevance of introducing the SRP in pre-service teachers training courses related to modeling activity at university are presented.

Keywords
Mathematical and Physical models, Study and Research Paths, pre-service teacher training, university level.

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State of the literature

- Many researches that address the problem of modelling in teacher education are recognized, but in this case, a co-disciplinary study of physics and mathematics is proposed.
- This research deals with co-disciplinary teaching based on questions from the theoretical framework of the Anthropological Theory of Didactic (ATD) since a Study and Research Path (SRP).
- A SRP was designed, implemented and analyzed in a state university, in a discipline that is part of the didactic studies within the Mathematics Training Course.

Contribution of this paper to the literature

- This research develops a co-disciplinary teach based on research teaching training in the pre-service teacher education at University level.
- It is not recognized, into the framework of Anthropological Theory of Didactic (ATD), researches which propos a SRP genuinely co-disciplinary. That is to say, those where both disciplines are studied on equally.
- We present the results of introducing a SRP during pre-service teacher training and the scope and limitations that the future teachers have developing the SRP.

Introduction
Recent researches emphasize that Mathematics teaching cannot focus on the transmission of knowledge from the teacher to the students. Some of these researches suggest providing students tools to study and inquiry "real" phenomena, intergrating Mathematics modelling in this process (Blue & Niss, 1991; Blomhøj, 2009; Barquero, Monreal & Ruíz-Munzón, 2018; Bosch, García, Gascón & Ruiz Higueras, 2006). The STEM approach (Science, Technology, Engineering & Maths) proposes an interdisciplinary approach to real-world problem solving, modelling, engineering and technology utilization (Chalmers, Carter & Cooper, 2017; Sanders, 2009; Czerniak & Johnson, 2014). Despite its importance, there is still no unified pedagogical framework for the design and implementation of STEM programs in school institutions (Heil, Pearson & Burger, 2013). Specifically, researches that deals with the joint study of Physics and Mathematics and the difficulties involved in studying these disciplines together is recognized (Blum & Leiß, 2007; Peña, Soto, Mariño, 2017; Taşar, 2010). Unlike the previous ones, our research proposes a co-disciplinary teaching, based on the framework of the Anthropological Theory of Didactic (ATD) (Chevallard, 1999, 2013, 2015). The Paradigm of Research and Questioning the World (Chevallard, 2013, Otero, et. al., 2013) advocates an epistemological and didactic revolution of the teaching of mathematics and school disciplines, which knowledge should be taught by its utility. These research propose a Study and Research Path as epistemological and didactic model (Chevallard, 2015), allowing a functional teaching of mathematics. The Study and Research Paths, also conceive modelling as an essential tool studying questions. In order to reach this aims, several changes in the traditional mainstream paradigm of teaching are necessary. This supposes changes of the syllabus and deeply transformations in how teachers manage and conceive knowledge to be taught, and in the roles of students and teachers regarding knowledge and the characteristics of the new paradigm.

This paper presents partial results of an exploratory research, obtained in two courses of pre-service mathematics teacher education (N=25) developing a SRP. The aim of the work is mainly to describe and to analyze how pre-service mathematics teachers deals with modelling and physical and mathematical models arousing from this process. The SRP starts from the generating

*Corresponding author
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question Q0: Why did the Movediza stone in Tandil fall? Which to be answered -in a provisional and unfinished way- needs the study of Physics and Mathematics jointly. The rationale of the paper is to describe the trainee teachers' activities and their difficulties when they must experience a SRP that it needs working with the physical and mathematical models to respond to a strong question. We intend answer the following questions: Which mathematical and physical models did the students develop during the Study and Research Path? Which constraints they faced developing and managing Mathematics and Physics models in the Study and Research Path?

Modelling and Study and Research Paths in the Anthropological Theory of Didactic (ATD)

The Anthropological Theory of Didactic defines the Study and Research Paths (SRP) as devices that allow the study of mathematics focusing on questions. According to the epistemology underlying a SRP, the starting points of mathematical knowledge are questions called generating questions, because its study should generate new questions so-called derivating. Teaching by means of SRP is complex and demands changes in the mathematical knowledge, in the roles of the teacher and students, and in general in the conformation of the entire didactical system. The developed Herbartian model (Chevallard, 2013) defines the components of the SRP:

$$\left[S(X;Y;Q) \rightarrow \{R_1^\lozenge, R_2^\lozenge, R_3^\lozenge, \ldots, R_n^\lozenge, Q_{n+1}, \ldots, Q_m, O_{m+1}, \ldots, O_p\}\right]$$

Where Q is a certain generating question; S is a didactical system around of the study of Q. S is formed by a group of people trying to answer the question (X) and by people helping the study (Y). In classrooms of mathematics, X represent the students and Y represent the teacher and other instruments helping in the search of answers to Q. S has to build a didactic medium M to study Q, whereas M is composed by different knowledge, expressed by $R_i^\lozenge$, Q_j and O_k. The $R_i^\lozenge$ are any existing answers or "socially accepted answers", the Q_j are derivating questions of Q, and the O_k are any other knowledge that must be studied developing the answers. Finally, $R^\heartsuit$ is some possible and partial response to Q given by S. In the a priori analysis stage, the specific and didactic knowledge, which could be involved within a SRP, is set up and the Praxeological Reference Model (PRM) is elaborated. The researchers analyze the potential set of questions, which the study and the research into Q might encompass together with the knowledge, mathematics and physics in this case, necessary to answer those questions (Chevallard, 2013).

SRPs are developed from a so-called generating question Q_0, because it does not admit an immediate response. It will be necessary to formulate derivating questions, and de-label the available answers. The didactic medium M is not built a priori, but from elaborating answers. Resources are incorporated when they are needed, at any time, under the condition that they have to be validated by the study community. The teacher directs the study process, but he doesn't have a preponderant role constructing M. The study group formulates and answers the questions, except the generating question, which is proposed by the teacher.

Methodology

This work involves a qualitative and exploratory research that aims to carry out a Study and Research Path (SRP), as proposed by the Anthropological Theory of Didactic (ATD), in a course of teacher training in mathematics at the University. The SRP was implemented in a state university, in the city of Tandil, Argentina, in a discipline that is part of the didactic studies within the Mathematics Training Course, in which two of the researchers are also teachers. Two implementations were carried out, in which took part N = 12 and N = 13 students of the last year (4th), aged between 21 and 33 years. The SRP was conducted in a total of 7 weekly hours provided in two lessons per week. In both implementations, three working groups were organized with approximately 4 members each.

It is important to note that these students had not studied physics at university, but they had a relatively solid mathematical foundation. In addition, the students had studied the Anthropological Theory of Didactic in two previous didactic courses. However, they report difficulties in understanding what a SRP is and how it works. In this sense, we propose to design, implement and analyze a co-disciplinary SRP of physics and mathematics, adapted to the institution in which it is developed.

As we have mentioned, the SRP is truly co-disciplinary in the sense that the interaction between physics and mathematics plays a central role and requires the study of both disciplines under equal conditions. In a SRP, the generating question Q_0 is proposed by the teacher, and this was done in the first lesson. Then, the students began their research in the library, selecting some texts, documents, etc. as possible $R_i^\lozenge$. In every class, each group presented and discussed with the teacher and the other groups their findings and possible ways to face Q_0. This study emphasizes on physical and mathematical models as the way to reconstructing an answer to problem of falling stone.

The Epistemological Model of Reference, the Study and Research Path and mathematical and physical models.

The starting question is Q_0: Why did the Movediza Stone in Tandil fall down? This enormous basalt stone has remained the city's landmark, providing it with a distinctive feature. Many local people and national celebrities visited the place to closely observe the natural monument. It was a 248-ton rock, sitting on the top of a 300-meter-high hill (above sea level), which presented very small oscillations when disturbed in a specific spot (**Figures 1, 2**). Unexpectedly, on February 28, 1912, the stone fell down the cliff and fractured into three pieces, filling the town with dismay by the loss of their symbol. For over 100 years, the event produced all kinds of conjectures and legends for the causes of the fall. Assuming that the fall can be explained by means of the Mechanical Resonance phenomenon, several questions Q_j emerged which are linked to the physical and mathematical knowledge necessary to answer Q_0.

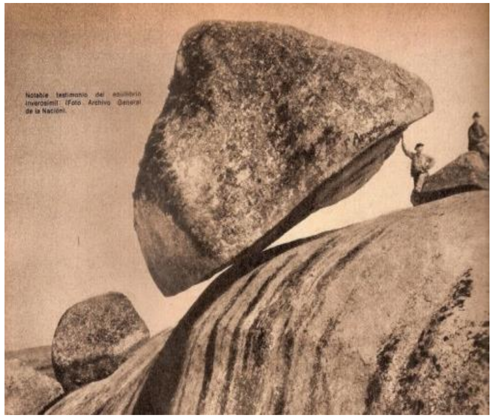


Figure 1. Photo Archivo General de la Nación Argentina.

Figure 2. Photo take from El Hage, E. (2007) "La piedra viva". Municipio de Tandil

If we consider that the stone was an oscillating system, the study can be carried out within the Mechanic Oscillations topic, starting from the ideal spring or the pendulum. In this case, frictionless systems are used, in which the only force in action is the restoring force depending (for small amplitude oscillations) in a linear way on the deviation respect to the equilibrium position. This model is known as simple harmonic oscillator whose motion, via Newton equations, is described by a second-order linear differential equation. Progressively, the system becomes more complex. If friction-produced damping is considered, it provides a new term to the differential equation connected to the first derivative of the position (speed). Finally, it is possible to study systems that apart from being damped, are under the influence of an external force, and therefore called driven systems. In the case that the external force is periodic and its frequency is approximately equal (the order of the approximation will be clarified later) to the natural (free of external forces) frequency of the oscillating system, a maximum in the oscillation amplitude is produced, generating the phenomenon known as mechanical resonance.

By increasing the complexity of the model, it is possible to consider a suspended rotating body, instead of a punctual mass. This leads to the study of the torque and the moment of inertia of an oscillating body. Here again, the linear system is for small amplitude oscillations and the damped and driven cases can be also considered, corresponding to the same mathematical model, but in which the parameters have a different physical interpretation.

However, as it refers to a suspended oscillating body, this is not a suitable physical model for the Movediza stone system. Since that, the base of the Stone was not flat, it is necessary to consider more precise models of the real situation. This leads to the mechanics of supported (and not hanging) oscillating rigid solids. In this case, we consider a rocker-like model in which the Movediza stone base is curved and it lies on a flat surface, where the oscillation is related to a combined translational and rotational motion (Otero, Arlego, Llanos, 2017). In **Figure 3**, we present the Epistemological Model of Reference from the point of view of the modelization it entails. We schematized the successive extensions of the physical models and the mathematical model which gets adapted to all of them, considering the parameters specificity. In this case, the intrinsic co-disciplinarity in the generating question entails a succession of physical models that can be described by the same mathematical model. These models, they describe better the real system. In figure, they are classified between "Oscillating hanging bodies" and "Oscillating supported bodies". In the first group they are considered the ideal pendulum model (IPM) when $\sin\theta \approx \theta$, i.e. the lineal model pendulum (LMP) and also the spring model (SM); for the harmonic cases, damped (LMP_D and SM_D) and force (LMP_F and SM_F); and the physical pendulum model (PPM). In the second group, the flat base model ($M_\blacktriangle$), the rigid solid model ($M_{R\text{-}S}$) and the deformable solid model ($M_{D\text{-}S}$).

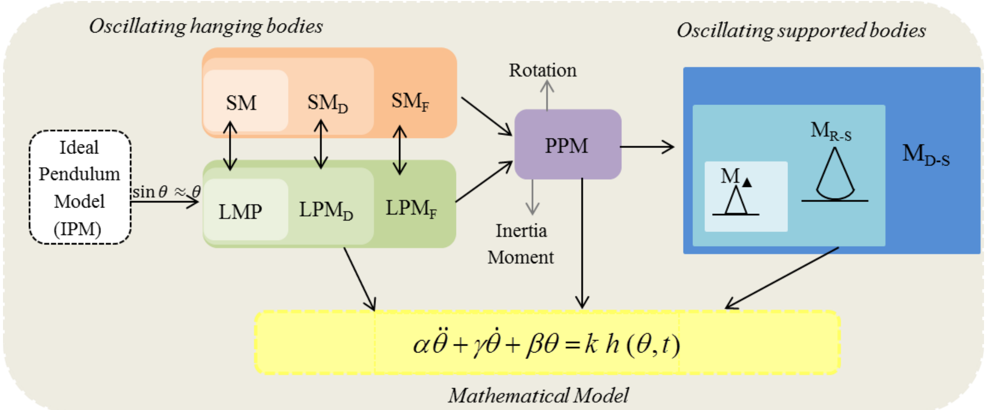


Figure 3. Sequence of models studied in RSC for increasing mathematical and physical complexity

The application of Newton laws to the rocker model of the stone leads to a differential equation where the parameters are specific of the Movediza system: mass, geometry, inertia moments, friction at the base, external torque, etc., which is given by the following *effective* Harmonic oscillator mathematical model of the Movediza physical system, called $M_{R\text{-}S}$:

$$\ddot{\varphi} + \gamma\dot{\varphi} + w_0^2\varphi = (M_0 / I)\cos(wt) \tag{1}$$

The stationary solution to equation (1) is: $\varphi(t) = \varphi_M \cos(wt - \psi)$; being the amplitude φ_M and the phase ψ

$$\varphi_M = \frac{M_0/I}{\sqrt{(w_0^2 - w^2)^2 + w^2\gamma^2}} \qquad \psi = tg^{-1}\left(\frac{\gamma w}{w_0^2 - w^2}\right) \tag{2}$$

The maximum of φ_M is for $w_m = \sqrt{w_0^2 - \frac{\gamma^2}{2}}$. The parameters: M_0 (external torque), I (inertia moment), w_0 (natural oscillation system frequency) and γ (damping coefficient), must be estimated. Detailed data about the shape, dimensions and center of mass position of the Movediza stone are available (Peralta et al. 2008) after a replica construction and its relocation in 2007 on the original place (although fixed to the surface and without possibility to oscillate). These data bring us the possibility to estimate some parameters in our model, as e.g. mass, inertia moment, and the distance of 7.1 m, from which the external torque could be exerted efficiently by up to five people (according to historical chronicles) to start the small oscillation. By using these values, it is possible to study the behaviour of the $\varphi_M(w)$ function for w_0 in a range of frequencies between 0,7 Hz and 1 Hz, historically recognized (Rojas, 1912) as the natural oscillation frequencies in the Movediza stone system and calculate for each case the maximum amplitude $\varphi_M(w_m)$.

The Stone would fall if $\varphi_c \le \varphi_M(w_m)$, being $\varphi_M(w_m) = M_0 / w_0 I\gamma$. Note that if γ is very small (as is expected to be in this case) we can neglect it from $w_m = \sqrt{w_0^2 - \frac{\gamma^2}{2}}$, leading to $w_m \approx w_0$. By using this approximation in Eq.(2) (left) the falling condition becomes $\varphi_c \le (M_0 / w_0 I\gamma)$.

The value of φ_c can be determined by an elementary stability analysis, which per the dimensions of the base of the stone and the centre of mass position is estimated to be approximately of 6° (Otero, et. Al., 2016). In the present model γ is a free parameter, for which we set "ad doc" a magnitude order $\gamma \ge 10^{-2}$. With this constraint, we find several situations, comprising different torques within the mentioned frequencies interval, supporting the overcoming of the critical angle, i.e., predicting the fall. Finally, in search a more appropriate approximation of the physics model for the damping that is clearly not due to air, we consider a more sophisticated model of the stone as a deformable solid ($M_{D\text{-}S}$), where the contact in the support is not a point but a finite extension, along which the normal force is distributed, being larger in the motion direction and generating a rolling resistance, manifested through a torque contrary to the motion. The rolling resistance depends on the speed of stone, giving a physical interpretation to the damping term. Therefore, the physics behind the damping is the same that makes a tire wheel rolling horizontally on the road come to a stop, but in the case of the stone, the deformation is much smaller. Although the deformable rocker model has extra free parameters, tabulated values of rolling resistance coefficient for stone on stone, allowed us to estimate and justify the damping values that we incorporate otherwise ad-hoc in the rigid rocker Movediza model (see Otero, Arlego and Llanos, 2017).

Data analysis

During the implementations, the students aim at answering how and why the stone fell down the cliff, different alternatives were explored about the causes of stone fall accepting among the hypotheses of the fall to the physical phenomenon of Mechanical Resonance, due to the repetitive action of an external agent, possibly several people (Holmberg, 1892; El Hage, Levy, 2012; Rojas, 1912). This gave rise to the study of oscillating systems. The trainee teachers' busily searched for an "already-made" mathematical and physical model, which allowed them to solve a differential equation in a specific way. Initially in both implementations, several physical and mathematical questions arose about the oscillations and resonance topics: How is an oscillation described? Which kind of oscillations are there? Which oscillation model would be the most appropriate to describe the stone? Which mathematical model should be used? Because it was a thoroughly new knowledge to them. The questions studied in the SRP around finding an answer to the fall of the stone are many, and a detailed analysis of them can be found in the work of Otero, et. al (2016).

The study group, divided in teams, provided different answers to the problem of oscillating systems (Alonso, Finn, 1992; Resnick, Halliday, Krane, 2001; Tipler, 1994; Elmer, 2011). The group G1 studied the harmonic oscillator (HO) (spring and simple pendulum), analyzing the solutions of the motion equations with GeoGebra. The group G2 analyzed the differences between the HO, forced, and damped oscillations in the spring case considering amplitude-time graphical representations. The group G3 considered the HO in the context of the physical pendulum. In all cases, they had difficulties to return to the original problem. Then, the teacher proposed filling the table showed in the **Figure 4**, as an instrument of synthesis.

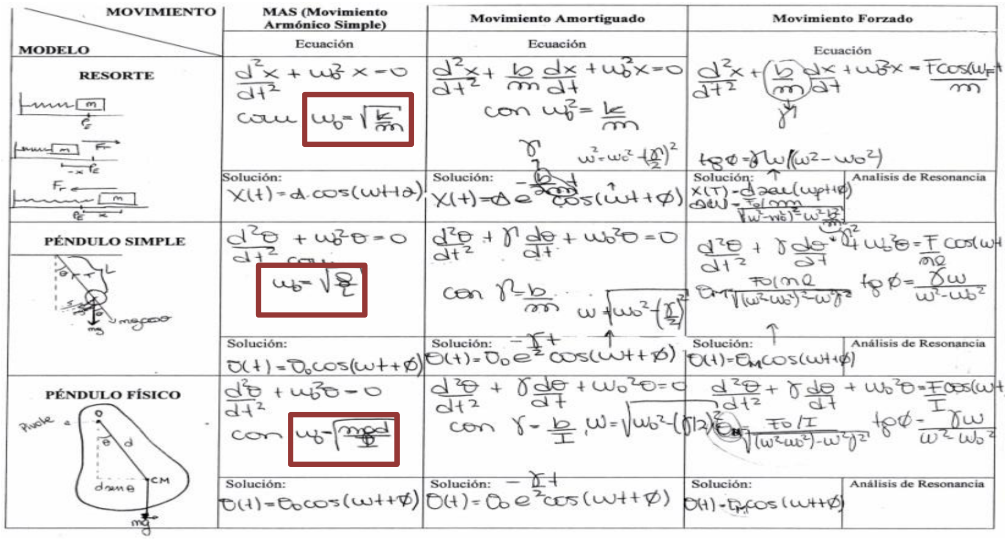
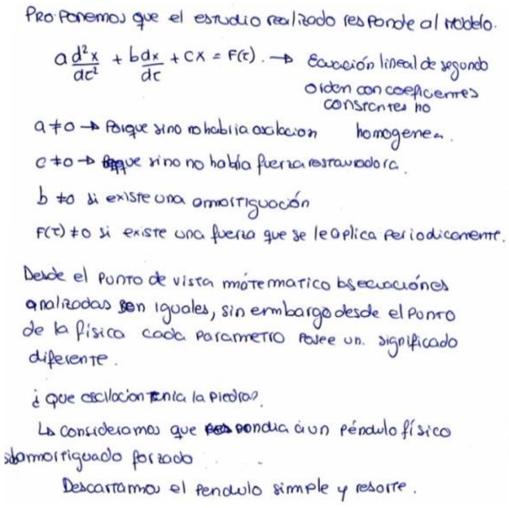


Figure 4. Protocol of the Group 3.

The teacher and the students analyzed the models and the meaning of the angular frequency *(w)*, the equation of motion and its solution for each case. Regarding damped and forced oscillations for the spring, the corresponding terms of the equation and the meanings of the parameters were also considered. Among the main results we highlight those showed in **Figure 5**.



The study performed responds to the model:

$$a \cdot \frac{d^2x}{dt^2} + b \cdot \frac{dx}{dt} + c \cdot x = f(t) \rightarrow$$ *Second order linear nonhomogeneous differential equations*

$a \neq 0$ → Because otherwise there would be no oscillation.
$c \neq 0$ → Because otherwise there would be no restoring force.
$b \neq 0$ → If there is damping.
$f(t) \neq 0$ Applying a periodically force.
Regarding mathematics the equations analyzed are similar, but however, from the physics point of view each parameter has a different meaning.
What oscillation does the stone have?

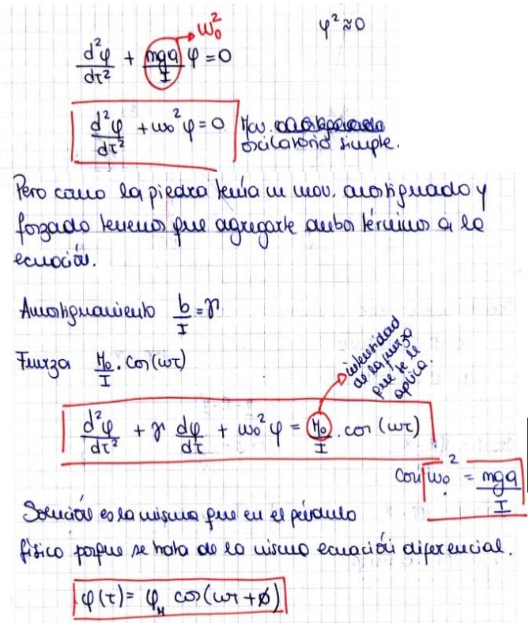
We consider that it would correspond to a forced underdamped physical pendulum.
We reject the simple pendulum and the spring

Figure 5. Protocol of the student 9 Group 2

Students noticed that the mathematical model is the same, but not the physical model, because the parameters represent different properties of the system. The students verified the solutions of the DE and they arrived at the solution provided in the physics textbooks, helped by a paper provided by the teacher. They studied the resonance condition and analyzed the amplitude function to determine the maximum. In this moment, some students presented strong objections to the possibility of using the physical pendulum model in the case of the stone, not so much in relation to a body that is supported but as an "inverted" pendulum. This directed the discussion towards the real system and the support basis, due models specifically referring to the system and they are no usually present in elementary textbooks, as it is the case of the rocked.

The teacher introduced the rocker model as a rigid solid combining the oscillating and rolling motion. While they analyzed the equation of motion, the students added the terms of damping and external force. **Figure 6** shows a synthesis of the models analyzed by the students. They considered the meaning of the angular frequency ($W_0 = \frac{mga}{I}$) concluding that the differential equation is similar however the parameters change.

Once the model was obtained, only the analysis of the parameters remained. The moment of inertia was calculated using the data provided by Peralta, et. al. (2008). The critical angle had already been calculated. The value $w_0 = 6{,}28hz$ was taken from Rojas (1912). Students failed using various values for the parameters, because they conceive them as fixed and unique. The main problem was to recognize the solution as a family of functions. Based on their questions, the teacher proposed to analyze this family of functions by means of spread sheets and graphics software, varying the different parameters. The students mainly proposed the use of GeoGebra, using sliders for M_0, γ, w_0. They emphasized the relevance of γ, determining the variation of the maximum amplitude for different values of γ. The students estimated γ to be the order of 10^{-2}, specifically between 0.01 and 0.02, depending on the people (2 to 5) considered. Finally, the students concluded that there is no a single set of parameters that support the fall of the stone, as originally they thought.



$$\frac{d^2\varphi}{dt^2} + \frac{mga}{I}\varphi = 0 \quad \rightarrow w_0^2$$

$$\frac{d^2\varphi}{dt^2} + \frac{w_0^2}{I}\varphi = 0$$ *Simple harmonic oscillator*

Since the Stone had a damped and forced motion, we should add both terms to the equation.

Damping: $\frac{b}{I} = \gamma$

Force: $\frac{M_0}{I} \cdot cos(wt)$ *Strength of force applied*

$$\frac{d^2\varphi}{dt^2} + \gamma\frac{d\varphi}{dt} + w_0^2\varphi = \frac{M_0}{I} \cdot cos(wt)$$

Con $w_0^2 = \frac{mga}{I}$

The solution is the same as the physical pendulum, because mathematically the differential equation.is the same.

$$\varphi(t) = \varphi_M cos(wt + \theta)$$

Figure 6. Protocol of the student 11 Group 3

Conclusions

In a SRP, students and teachers integrate the study community facing together situations of study and research. In both implementations, the pre-service training teachers studied physics and mathematics thoroughly and showed a good disposition to deal with questions they had never considered before. It is important to highlight the role of the teacher in the SRP. For the teacher, the question Q_0 was also an open question, for which, did not have any a priori closed answer. In this sense both, the students and the teacher took a genuinely active part in the SRP. This device is very appropriate to foster interdisciplinary study, because it allows studying only the necessary mathematics or physics to answer a question, returning to the original problem. However, it is not only important to decide what content to study, but how to use them, and so the physical and mathematical models and their rationale emerge.

The construction of a possible answer to the generating question Q_0, driven the study and the analysis of several physic models related to oscillating systems like springs, single pendulum and physical pendulum, including damped and driven oscillators. However, none of these physical models were adequate to the stone. By reanalysing the real system in a more detailed way, students established that the previous models did not describe some essential aspects of the stone, the most important being the fact that the real system is an object supported on a surface and that is not hanging, like the previous physical models. Then, in the search for a reason to make a supported physical stone model oscillate, the hypothesis that the contact surface between the stone and the base is not flat, but some of the two or both have a certain curvature, emerged, something that in fact had some historical evidence. In this way, the physical model of the rocked arises as the most appropriate to describe the oscillations of a supported object.

The main obstacle for the trainee teachers is epistemic, they consider implicitly to mathematic as pure. Another epistemic obstacle is the absence of modelling as an activity own of mathematics. Future teachers conceive algebraic modelling as a single application, so they do not understand the role of families of functions as solutions of differential equations and that variables and parameters are not permanently fixed. In the beginning, the most relevant obstacles pre-service teachers faced were related to the physics knowledge, insofar as the physical model was sophisticated and the physical knowledge necessary to treat it was insufficient. However later, the main difficult consisted how to use algebraic-functional modelling involved in the solution of the differential equations. Notwithstanding, we can say that some aspects of the modelling processes in the sense of the Anthropological Theory of Didactic (Barquero, Bosch and Gascón, 2011), were accomplished. The SRP evidences the inadequate of the available answers to treat the motion of the stone, and the complexity of the models required to study Q_0. Finally, the research showed that mathematics teaching for pre-service teachers at university level has to change, in order to provide a different epistemological vision about mathematics and its uses. To learn mathematics is necessary to make mathematics, modelling is an essential part of mathematical activity that cannot be out of the mathematics teacher's formation in any discipline.

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