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Modeling the Mole Understanding with Mathematical Reasoning

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Abstract

The amount of substance, expressed in the units of moles is an essential concept in chemistry and physics. Students entering physics courses usually possess a chemistry background. However, this study showed that their understanding of units of matter on the microscopic level is fragile, and needs improvement. Research shows that the complexity of interpretations of quantities expressed as ratios; molar mass or atomic mass makes formulating a dimensional analysis or proportion of these ratios unclear to students. Based on these findings, this study proposes applying equations of fundamental constants and proportional reasoning, instead of ratios, as the main building blocks to formulate conversion algorithms. In the line of that, a deductively designed lecture was delivered to a group (*N*=25) freshman college physics students. While on the pretest, only (*N*=4, 16%) correctly converted a mass of a substance expressed in kilograms to a number of moles, on the posttest the percentage of correct answers increased (N=20, 80%) suggesting that proportional reasoning coupled with fundamental constants brings clarity to the process and improves its understanding.

Keywords

The mole, Conversions, Proportional reasoning, Thermodynamics, mathematical modelling, instructional model, mole understanding

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Introduction

A diversity of research ranging from a perspective of psychology learning through historical and philosophical views (Niaz, 1989; Furio et al., 2002; Siswaningsih et al., 2017, Feb.) had identified several deficiencies associated with an understanding of the mole; (a) students often identify the mole with mass, volume, and Avogadro's number of elementary entities; (b) students avoid inserting the abbreviation of the amount in the units of moles, (c) students confuse *molar* mass with *molecular* mass. Other studies (Dahsah and Coll, 2007; Musa, 2009; Fach et al., 2006) showed students' weak skills of handling conversion problems involving the mole, mainly, when applying the meaning *amount of substance.* Johnstone (1971) found out that students difficulties with the concept of the mole are widespread and are rooted even in the SI definition of the mole. In this line, Fang (2014) suggested that "the research of the past 40 years is that the way the mole is conceptualized in educational settings is inconsistent with the meaning of the mole expressed in the SI definition" (p. 351).

Multiple attempts have been made to identify the causes of these difficulties and help students understand the concept of the amount of substance and consequently the processes of conversions. Uce (2009) used a conceptual change method to develop the mole understanding. Bunce (1994) claimed that the problem is rooted in inappropriate teaching strategies. Staver and Lumpe (1995) pointed out that the difficulties originated in an insufficient understanding of the algorithmic routes applied during conversions. Similarly, Tullberg et al. (1994) found out that educators teach the concept of the mole using extensive reasoning methods instead of direct algebraic equations. Schmidt (1990) argued that before introducing the concept of mole, students must realize that different atoms have different atomic masses. Niaz (1987) suggested a closer science and mathematics integration to introducing the idea of the mole. In a similar vein, Soon et al. (2011) found out that students do not transfer their algebraic skills to their science automatically and suggested a modeling approach to intertwine mathematical tools with science concepts. Concurently, Whelan (1977) reached out to educators and called for a closer consistency of teaching the concept of mole with the methods presented in the textbooks. More recently, Fang et al. (2014) suggested a map concept as a framework for teaching the concept of mole and the conversions. They proposed using large rectangular shapes to depict schematically the mole and smaller rectangles inserted inside of the larger to represent sub-concepts such as molar mass or atomic mass. Indriyanti & Barke (2017, August) suggested introducing the mole concept by having students count the number of items using weighing. The idea of weighting was employed to have students realize that it is not possible to count the very large number of particles. Thus the unit of the mole was introduced. This idea of the mole conceptualization was earlier exercised by Dominic (1996).

Milton (2009) suggested introducing a different, more intuitive, definition of the amount of substance that measured the size of an ensemble of entities (atoms, molecules, ions, electrons, other particles, or specified groups of particles). Scott (2012) concluded that students are deficient in their mathematical skills and advocated for improving communication between science and mathematics departments to eliminate this deficiency. This study could be considered as a response to that call. Erceg et al., 2016) identified several students' misconceptions about the kinetic molecular theory of gases that could be accounted for a lack of understanding of units describing the mass of gas.

While all findings provided a wealth of suggestions on targeting various weaknesses of the mole understanding, each pinpointed a specific aspect of the teaching process rather than suggested a more general method. This study will attempt to encompass all of these suggestions especially these suggested by Soon et al. (2011) and Scott (2012) who advocated for more integration between algebraic operations and algorithms applied during conversions.

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Foundations of the Theoretical Framework

Summary of students' deficiencies extracted from the prior research

Prior research findings helped to formulate the test instrument for this study, and consequently, they helped formulate the instructional unit (intervention). All prompts generated nine suggestions that were summarized in **Table 1**.

Table 1. Prompts for formulating the theoretical framework

Challenges with the mole understanding	Suggested interventions
1. Overemphasized conceptual approaches during conversions.	Using the structure of a proportion and equations throughout various types of conversions.
2. High diversity of units used to describe the amount of quantities at a microscopic level.	Introducing the mole as a basic unit of amount of substance in the form; $1mol = 6.02 \times 10^{23}\ atoms$
3. Lack of systematization of the different units of measuring matter.	Introducing *molar mass* prior to introducing *atomic mass.*
4. Overemphasizing the formal definition of the mole as regarded to Avogadro's number and the number of particles in carbon-12.	Introducing the formal definition of the mole after students are fluent in basic conversion techniques.
5. Lack of contrasting traditional units of mass (kg) with the amount of mass expressed in moles.	Providing students with opportunities for contrasting various units of mass and embrace it with conversions.
6. Lack of parallelism to algebraic operations while setting up the converting the units of mass, e.g., expressed in kg to moles or a number of particles.	Using the idea of constructing *proportion* and the periodic table of elements to find reference molar and atomic masses.
7. Difficulties with interpretations of phrases like *molar mass* or *atomic mass* because the equivalent descriptions (e.g., *kilogram mass* or *pound-mass*) are not used in sciences.	Using the phrase *mass of 1 mole* instead of *molar mass* and *mass of 1 atom* instead of *atomic mass* and consequently apply these statements to build proportions.
8. Diminished effect of algebraic manipulations due to applied detailed dimensional analysis.	Replacing the process of dimensional analysis and simplifying the units using regular algebraic algorithms for terms' reduction.
9. Lack of connection between atomic mass representing the mass of 1 mole of a substance.	Emphasizing *mass number* (periodic table) as representing the mass, in grams, of 1 mole of a substance.

These recommendations guided the conceptual design of the instructional unit. Following these recommendations, algebraic structures for the intervention were selected. The review of the theoretical foundations of these structures; proportions, rates, and ratios follows.

Proportional Reasoning, Rates, and Ratios

The structure of a proportion along with its interpretations and limitations will serve as the primary algebraic structure to develop students' algebraic reasoning that should help them to support conversions. Thus an outline of the methodology of constructing proportions as seen through the prism of mathematics but enriched to model to support scientific quantities follows. This section can be considered background that is borrowed from mathematics to quantify chemical quantities. A similar concept called the laws of proportions (Mansoor & Rodrigues, 2001) can be found in the literature. This study introduces the learners to the modeling approach of enacting and solving proportions departing from bringing forth the theory of equation, fundamental constants, and their scientific interpretations. Concise structures of mathematics will support quantification processes, and corresponding scientific embodiments will be derived from the outputs of the computations.

Proportional reasoning is foundational to many high school and higher levels of mathematics (Confrey, 2008) and it is commonly referred to as the capacity to compare the values of two quantities within the same system. The conceptual underpinning of proportion is usually developed in junior high school (e.g., see Fielding-Wells et al., 2013) and the applications are used high school and higher-level mathematics, for example, to build differential equations or to algebraically model dependence of two quantities. Research has shown that proportional reasoning is more stable with maturity and experience (Clark and Kamii 1996). Therefore its more formal structure (Coffield, 2000) is suggested to be used.

Proportional reasoning denotes reasoning with two quantities (variables) between which there exists a linear functional relationship of the form $y = mx$ where y and x represent the quantities and m represents a proportionality constant often expressed in the form of a constant rate or ratio. The proportionality constant, m, can be visualized as the slope or inclination of the a corresponding linear function $y = mx$. The functional representation of proportionality is often regarded in mathematics as a dynamic proportionality (Miyakawa & Winsløw, 2009) because the function can be used to find many values of the dependent or independent variable if a constancy of their rates or ratios within a certain domain is established.

The other form of proportionality more often applied in problem-solving in mathematics that will be used in this study is called a static proportionality or a specific set up that can be used for computing a unique value. Static proportionality embeds two different situations within the same system of variables. This form of proportionality will dominate in this paper; thus, its more in-depth analysis is discussed. Static proportionality or proportion is an equality of two rates or two ratios usually with one unknown quantity. Since rates and ratios will also be implemented in the instructional unit, let's also review them. What is the difference between ratio and rate?

a) The ratio is a quotient of two similar/homogeneous (with the same dimensions/units or dimensionless) quantities. For example; $\frac{12\text{m}^2}{6\text{m}^2}, \frac{1\text{ kg}}{2\text{kg}}, \frac{3\text{L}}{8\text{L}}$, etc.

b) The rate is a quotient of two dissimilar/heterogeneous (with different dimensions/units) quantities. For example; $\frac{7\text{m}^2}{6\text{s}}, \frac{20\text{kg}}{5\text{m}^3}, \frac{3\text{mol}}{6\text{L}}$, etc.

Ratios and rates can be reduced (or simplified by division) and then the resulted magnitude represents a unit ratio or rate called the proportionality constant. For example, $\frac{12\text{m}^2}{6\text{m}^2} = 2$, $\frac{20\text{kg}}{5\text{m}^2} = 4\text{ kg/m}^2$. It is to note that in this process, ratios might become dimensionless, however, rates retain the units. Both rates and ratios can lead to formulating static proportionalities called proportions.

How to convert a dynamic proportionality into a static one or vice versa? Consider $y = mx$ and $m = \frac{y_1 - 0}{x_1 - 0} = \frac{y_1}{x_1}$. Let's substitute $\frac{y_1}{x_1}$ for m in $y = mx$, thus $y = \frac{y_1}{x_1}x$. Since a static proportionality refers to unique values, then let's further replace y by y_2 and x by x_2 and rearrange the terms;

$$\frac{y_2}{x_2} = \frac{y_1}{x_1} \quad (1)$$

Statement (1) represents a static proportionality of two rates. By applying a cross multiplication and division, this proportion can be converted to a proportion of two ratios respectively

$$\frac{y_2}{y_1} = \frac{x_2}{x_1} \quad (2)$$

A static proportion, either containing two rates or ratios, can be formulated only for quantities satisfying a condition for a direct proportionality. For example the proportions; $\frac{20\text{kg}}{5\text{m}^3} = \frac{x}{4\text{m}^3}$, or $\frac{4\text{m}^3}{5\text{m}^3} = \frac{x}{20\text{kg}}$ represent direct proportionality because, from algebra point of view, a corresponding dynamic representation will lead to a form $y = mx$. It is usually up to the students whether they chose equivalence of two rates or two ratios to construct a proportion and solve it. Can a static proportionality be formally converted into a dynamic one? Yes, if one of the sides of the proportion contains a rate or ratio and their respective variables, a dynamic proportion can be formulated. For example if $\frac{20\text{kg}}{5\text{m}^3} = \frac{y}{x}$, then $y = \left(4\frac{\text{kg}}{\text{m}^3}\right)x$, where y represents kilograms and x represents meters[3].

In the function representation; $y = (4\frac{\text{kg}}{\text{m}^3})x$, the rate, $4\frac{\text{kg}}{\text{m}^3}$, takes earlier mentioned and more sophisticated interpretation as the inclination of the function, known also as slope that is equal to $\frac{\text{rise}}{\text{run}}$. Understading proportionality extends beyond setting up proportions and computing the value of the missing quantity; it is linked with one of the most fundamental function in algebra; a linear function. Realizing its limitations should enhance its applications as regraded the domain and scope of this research.

Method

This undertaking can be classified a pretest-posttest one group experimental study (Shadish et al., 2002) with predominantly qualitative analysis of its results. While having a control group would support the validity of the research, furnishing one was not possible, thus one group design was used and followed. The instructional unit was developed to address students' deficiencies in handling the mole understanding and its subsequent conversions that were located in the prior research findings and also the pretest designed for this study.

The purpose of the pre-test took a different aim from the traditional; it served as an additional source of prompts to design the pedagogy of the instructional unit. The posttest was used to assess students' understanding of the unit of mole and their ability to adapt proportional reasoning to conversions and other processes that involved the unit of the mole.

This study was conducted with a group of 25 (10 females and 15 males) freshman college physics students who did have a prior background about the mole and its conversion from high school chemistry and physics courses. Most of the students (*N*=18) were majoring in engineering programs and the rest in other like environmental studies. Majority of these students (*N*=20, 80%) were Caucasian, and the rest (*N*=5, 20%) consisted of other races. The students graduated from various high schools. All took part in the study voluntarily.

Research question

The following question guided this study:

Can students adopt the concept of proportion as an algebraic tool to convert and develop an understanding of the mole as a unit of substance measure?

While the prior research findings provided insights into how chemistry students perceive the idea of mole and its applications, there was a need to assess their strengths and weaknesses in this domain and find out if they can adopt different methods of handling similar tasks. Thus, the pretest, which is traditionally used to support the learning effects computations, in this study was also used to guide the theoretical design of the treatment.

Lecture component

The instructional unit took a form of discovery type lecture. The instructor posited questions, asked students for their inputs and then together with the students formulated proportions and proved or disproved students claims. While the unit could be devoted to conceptualizing the idea of the mole, a more pragmatic approach was taken to include not only the interpretation of the mole as an amount of substance but also to merge it with developing conversion techniques. It was assumed that providing students with opportunities to apply the definition will not only enhance the conceptual understanding but also illustrate a high diversity of conversion types and offer techniques for handling them. The instructional unit was designed to last for a typical one class period (55min), and it consisted of several segments organized deductively. It began with more general conversions and zoomed gradually into more detailed once. The segments were initiated by conceptual questions whose purpose was to integrate students' conceptual understanding before immersing in more formal algebraic conducts.

Mole as a basic unit of amount of substance

The idea of the mole is challenging to understand because it does not contain the unit of mass such as kilograms or grams even though it measures the amount of substance. The instructor initiated the lecture by supporting the need for the mole introduction to science and stated that on a molecular level a sample of a known substance could be characterized by its mass (in kg) whereas on a microscopic level the sample can be characterized by the number of its entities. These are atoms in the case of most elements. Samples of the order of milligrams, or less, are needed for microscopic experiments. They contain a vast number of atoms, let us say, of an order 10^{30}. Hence, counting atoms as single entities were impossible, and a unit representing a more substantial amount was needed. The unit, convenient for calculations and laboratory practice, consisting of 6.02×10^{23} atoms were called *the mole*, and the base quantity of which the mole is the base unit bears the name *amount of substance*. If there are 6.02×10^{23} atoms in a container, then one can say that there is 1 mole of the atoms in the container. The teacher assured that a more formal definition of the unit of 1 mole would be provided later. The teacher also pointed out that while the *amount of mass* must be expressed in the units of mass (grams or kilograms etc.), the phrase *amount of substance,* that does not contain the term *mass,* does not induce the units of grams or kilograms. Thus, the unit of the mole does not explicitly show the amount of mass traditionally expressed in kilograms.

While the mass of a substance is usually denoted by a lower-case m, e.g., $m = 4\,kg$, the amount of substance is denoted by n, e.g., $n = 10\,moles$ that is abbreviated to $n = 10\,mol$. To further conceptualize the idea of the mole, the teacher added that while the majority of fundamental physical quantities could be measured using devices; e.g., temperature by a thermometer or electric current by the ammeter no device would measure directly the *amount of substance (in moles)*. However, by applying algebraic operations, a substance expressed in kilograms can be converted to moles and vice versa. The ultimate question was how to convert the mass of a substance to a corresponding number of moles? Before immersing into mass - mole conversions, the teacher introduced a technique of converting a number of particles to moles to support conceptual understanding of the idea of 1 mole and to highlight the mechanism of formulating a proportion. These conversions were to have students realize that the unit of the mole can be used to link the micro- and macro-worlds and vice versa.

Converting the number of atoms to the units of moles and vice versa

The teacher posited the following question: Does the number of moles of a substance depend on the number of atoms? After a short discussion confirming the answer, he posted a problem.

Example 1. Convert 12.04×10^{23} atoms of silver to moles.

To convert the number of atoms to moles, the teacher labeled the variable of interest as *n* represented by $12.04 \times 10^{23}\,atoms$ and demonstrated the process of formulating two parallel statements that will lead to making up a proportion; first containing fundamental constant called for the purpose of the study *known statement*; $1mol = 6.02 \times 10^{23}\,atoms$ and the other containing the variable of interest;

Known statement: $1mol = 6.02 \times 10^{23}\,atoms$

A statement with a variable: $n\,moles = 12.04 \times 10^{23}\,atoms$

While the above setup of equations involved two algebraic operations to be solved as opposed to one if the *statement with a variable* were placed on the top and the *known statement* below, this arrangement was intentional to highlight the modeling process. The construction of all of the proportions during this lecture was consistently initiated from writing down the *known statements*. The instructor explained that the order of the statements does not affect the final answer and practically it will be up to the students what order they prefer. The instructor also pointed out the parallelism in formulating these statements; both contain moles on their left sides and atoms on their right sides. By embracing the statements in the algorithm of division, a proportion of two ratios was formulated;

$$\frac{1\,mol}{n\,mol} = \frac{6.02 \times 10^{23}\,\cancel{atoms}}{12.04\ \times 10^{23}\,\cancel{atoms}}$$

By cross multiplying, canceling the units of atoms and solving the proportion for *n*, the students learned that

$$n = \frac{(1mol)(12.04 \times 10^{23}\,atoms)}{6.02\ \times 10^{23}\,atoms} = 2\,mol$$

Some students had figured out the answer prior solving the proportion, yet the purpose of writing the statements was to have them understand the mechanism of setting up the proportion. By initiating the proportion by using fundamental statements rather than ratios, the students were to realize the foundation of the structure of the proportion. To enhance further the mole concept as connecting the macro-world with the micro-world, the teacher posited a further question:

Can amount of a substance expressed in moles be converted back to a number of atoms?

After a short discussion with the students, he wrote example 2 on the board.

Example 2. How many atoms of monoatomic hydrogen are there in 4.2 moles of the substance?

The teacher pointed out that the goal is to use a similar idea of proportion to solve the problem. Let x represent the number of atoms in 4.2 moles. The teacher did not use n to label the number of atoms to avoid confusion with n denoting the number of moles. It is that almost all students who applied proportions on the posttest used x to denote any variable of interest.

Known statement: $1mol = 6.02 \times 10^{23}\ atoms$

A statement with a variable: $4.2\ moles = x\ atoms$

These statements lead to the following proportion of two ratios:

$$\frac{1\ mol}{4.2\ mol} = \frac{6.02 \times 10^{23}\ atoms}{x\ atoms}$$

That solved produced $x = 25.28 \times 10^{25}$ atoms.

After solving a couple of more examples of a similar type, the teacher invited the students to determine if the type of substance expressed in a number of atoms affected the corresponding amount expressed in moles. By observing patterns of formulating the proportions (that did not include the atomic mass of the atoms), students realized independence of the properties of the unit of the mole from the atomic mass of the substance. Students noted that mole-atoms conversions could be embraced in a dynamic proportion; $\frac{1mole}{n} = \frac{6.02 \times 10^{23}\ atoms}{x}$ that lead to its more general version applied in the dimensional analysis; $x = \left(6.02 \times 10^{23} \frac{atoms}{mole}\right) \cdot n$, where x represented the number of atoms, and n the number of moles. A frequent use of $6.02 \times 10^{23}\ atoms$ prompted an introduction of the Avogadro's number. After assuring that the students were comfortable with mole-atoms conversion, the instructor proceeded to phase 2 that dealt with mass - moles conversion.

Expressing mass of a substance in kilograms in terms of moles and vice versa

This type of operation dominates in science, and it involves a reference to the periodic table of elements. Due to a low percentage of correct answers on the pretest (32%) and suggestions of the prior research, this path of conversion required a more detailed approach. To alert the students that if substance mass in kilograms is given, then the number of moles will depend on the atomic mass of the substance, the teacher posited a conceptual question:

Does 2 kg of mercury $^{200.59}_{80}Hg$*, contain the same number of moles as 2 kg of potassium,* $^{39.102}_{19}K$*? Is it necessary to use Avogadro's number during the process of converting these quantities?*

Recalling their prior knowledge, most of the students claimed that the number of moles would not be the same and suggested that Avogadro's number be included in the proportion along with the mass of 1 atom of each of the substances. Once this was known, the number of atoms could be found by setting up a proportion like in phase 1. Overusing Avogadro's number was reported by the prior research as students' weaknesses. Thus, while this was a correct approach, the students, guided by the teacher, eventually realized that the Avogadro's number would be used twice in two inverse algebraic operations; multiplication and division, thus it would produce a composite of inverse operations that yields an identity operation. The teacher redirected the students thinking toward interpreting the *mass number* as a mass, expressed in kg, of one mole of a substance. He reviewed the interpretations of numbers associated with atoms as described in the periodic table of elements, $^{A}_{M}X$, where *A* is called *mass number*, and M is called *atomic number*. Mass number, as given in the periodic table of elements, is expressed in terms of *u* the atomic mass unit that equals to $1.66 \times 10^{-27}\ kg$. In conversion problems, *mass number* can take a dual interpretation (see **Figure 1**); it can be interpreted as a *mass of one mole of atoms* of a substance (called also *molar mass* and expressed in grams), or a *mass of one atom* of the substance (called also *atomic mass* and expressed in kilograms when multiplied by the atomic mass unit *u*). Both interpretations are mutually dependent, and one can be converted into the other or used as indicated by a specific task. Understanding of the differences between these two different interpretations will be exemplified in the conversion problems that follow.

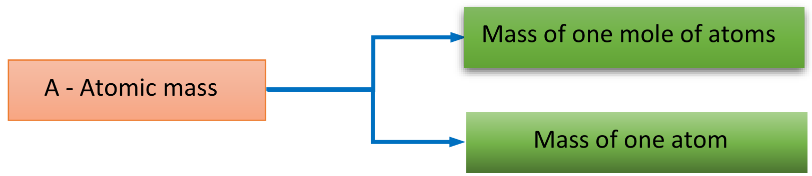

Figure 1. Dual interpretation of the atomic mass number suggested in conversions

The teacher informed the students that during this phase, they would use the mass number as representing the mass, in grams, of one mole. After that inclusion, the teacher referred to the problem stated and pointed out the positions of the elements in the periodic table while asking the students to predict the answer before applying the formal process. The phase of predicting was to have students further conceptualize the idea of 1mole which when coupled with the interpretation of notations used in the periodic table provided a bridge to traditional mass units. Students were ready now to construct proportions and convert both substances to moles.

Example 3. Prove that 2 kg of mercury, $^{200.59}_{80}Hg$, contains a different number of moles than 2 kg of potassium, $^{39.102}_{19}K$?

Referring to an established earlier structure for building a proportion, the teacher labeled n as representing the number of moles in 2 kg of mercury.

Known statement: $1\text{mol} = 200.59\text{g}$

A statement with a variable: $n\text{ mol} = 2000\text{g}$

$$\frac{1\text{ mol}}{n\text{ mol}} = \frac{200.59\text{g}}{2000\text{g}}$$

Thus $n = 9.97$ mol. By labeling the variable and setting up a similar proportion for potassium, the students learned that for potassium, $^{30.102}_{19}K$, $n = 66.44$ mol. Students confirmed that their predictions were correct that potassium with a lower mass of 1 mole of the substance generated more moles. While the number of moles was independent of the type of substance when given by the number of atoms, the number of moles depended on the type of substance when the substance was given by its mass.

Expressing mass of a substance in terms of the number of atoms and vice versa

This type of operation was designed to find connections between phase 1 and phase 2 and practice using the mass number as one representing the mass of one atom.

Example 4. How to set up a proportion to find the number of atoms in 4kg of copper?

Following the structures of previous conversions, the students suggested to apply two steps; first to convert the mass to moles (as in phase 2) and then by using the property that $1\text{mol} = 6.02 \times 10^{23}$ atoms (as in phase 1) find the number of atoms. This path of thinking showed that they understood the basic conversion techniques and the technique of setting up the proportions. After confirming the answer, the teacher intended to expand the type of conversions and suggested another way that used the mass number as a mass of one atom of the substance expressed in kilograms. Referring to the periodic table of elements, one learned that copper is described as $^{63.54}_{29}Cu$. How to construct a proportion that would use the known statement of the mass of one atom of copper?

Let x represent the mass of one atom of copper;

Known statement: $1\text{ AMU} = 1.67 \times 10^{-27}$ kg

A statement with a variable: $63.54\text{ AMU} = x\text{ kg}$

Creating a proportion and solving it for the variable, $x = 1.06 \times 10^{-25}$ kg that showed the mass of one atom of copper. This process did not provide the answer for the initial question yet, and another proportion was needed. Let x represent the number of atoms in 4 kg of copper.

Known statement: $1\text{ atom} = 1.06 \times 10^{-25}$ kg

A statement with a variable: $x\text{ atoms} = 4\text{ kg}$

By solving the proportion, one learned that $x = 3.77 \times 10^{25}$ atoms, which represented the number of atoms contained in 4 kg of copper. While the process depicted all the details, most students simplified it by formulating the proportion $\frac{1}{x} = \frac{1.06 \times 10^{-25}\text{kg}}{4\text{kg}}$ or just writing the final step $x = \frac{4\text{kg}}{1.06 \times 10^{-25}\text{ kg}}$ which was also accepted.

After solving these examples, the students were invited to solve textbook problems and design a theoretical experiment that would lead to formulating a regression line whose slope would represent the unit of the mole. The purpose of asking students to design an experiment was to have them link static and dynamic representations of proportions and get them more fluent in using both. During a next lecture (not discussed in this study), the teacher introduced a formal definition of 1 mole, made further connections to Avogadro's number and carbon-12 and used the conversion techniques to solve problems regarding ideal gas law; $pV = nRT$ and $pV = Nk_BT$, where p represented pressure inside the gas expressed in pascals, V volume of the gas expressed in meters cubed, n number of moles, $R = 8.31\frac{J}{mol} \cdot K$ gas constant, T gas temperature expressed in Kelvin scale, N number of atoms, and $k_B = 1.381 \times 10^{-23}$J/K Boltzmann's constant. While working on converting, students were given options of using any methods of their choice.

Data Analysis

Analysis of the pretest results

Three questions, designed by the author and consulted with science professionals were used to assess the students' current understanding of the unit of substance; Item #1 that was a True/False type, Item #2 that required the students to convert given mass to number of moles, and Item #3 that assessed students' correct understanding of the definitions of 1 mole, molar mass, and atomic mass. **Table 2** shows descriptive analysis of Item #1.

The percent of correct answers on Item #1, part #a was relatively high (88%) which showed that the students understand the unit as the quantity measuring substance. The percent of correct answers on the remaining parts of this question varied. A rather low percentage of the correct answers (32%) was reported on the interpretation of the molar mass (#1c). This deficiency was addressed while formulating the theoretical framework.

Table 2. Students' pretest responses (Item #1, True/False)

Multiple Choices	Answer/Percent correct
a) The mass of two moles of oxygen is the same as the mass of two moles of nitrogen	False, 88% (*N*=22)
b) Two moles of iron contain the same number of atoms as two moles of zinc.	True, 56 % (*N*=14) True, 32 % (*N*=8)
c) Molar mass can be expressed in kilograms or grams.	True, 72 % (*N*=18)
d) Both atomic mass and molar mass can be used to find the mass of a certain number of particles.	

Item #2 asked the students to convert 2 kg of sodium $^{22.99}_{11}Na$ to moles and it was correctly solved by 20% (*N*=5) of the students. About 24% (*N*=6) of the students did not attempt to solve it, 15% (*N*=4) got lost in setting up the conversion process (dimensional analysis). The remaining 39% (*N*=10) inserted the Avogadro's number in the dimensional analysis and could not carry out the units' cancellation. None of these students attempted to apply proportional reasoning to solve the problem.

Item #3 asked students to define the unit of the mole, molar mass, and atomic mass. These students showed that they understood the conceptual definition of the mole; the majority of them (80%, *N*=20) stated that mole represents the amount of particles/atoms/entities a substance has or an object is made of. The remaining 20% (*N*=5), used more general descriptions. Verbatim, for example: "It is used to describe matter and can be converted to other things," "a unit of measure to express mass, volume, and particles," "a basic unit used for mass." However, the students did not equally succeed on conceptualizing the phrases; molar mass and atomic mass on which the percentages of correct answers were respectively (25%, *N*=6) and (30%, *N*=8). The pretest questions were not returned to students, nor they were discussed until the study was completed.

Summary of the pre-test and post-test results

The students took the posttest after about six weeks from the date they took the pretest. The format of the posttest questions was like the one given on pretest. A summary of pre-test/post-test responses to Question 1 are included in **Table 3**. The posttest results showed an improvement in all areas tested and especially in interpreting molar mass (#1c) that might be accounted for highlighting established verbal definition as the mass of one mole. This result suggests that the commonly used combination of the words molar and mass does not clearly describe the physical unit of the quantity. In a similar rate (80%, N=20) the students succeeded in converting 2 kg of sodium, $^{22.99}_{11}Na$, to moles. The results on both questions supported the hypothesis that using the structure of proportion and embracing the process in a consistent and systematic procedures, helped students understand the interpretation of the unit of the mole and learn the conversion techniques. Percentages of students who correctly interpreted molar mass (86%, *N*=22) and atomic mass (90%, 23) was also higher on the posttest.

Table 3. Students' pretests and posttest responses to Question #1.

Question: Classify as True/False	Pretest	Posttest
a) The mass of two moles of oxygen is the same as the mass of two moles of nitrogen	False, 88% (*N*=22)	90% (*N*=23)
b) Two moles of iron contain the same number of atoms as two moles of zinc.	True, 56% (*N*=14%)	85 % (*N*=21)
c) Molar mass can be expressed in kilograms or grams.	True, 32% (*N*=8)	78 % (*N*=20)
d) Both atomic mass and molar mass can be used to find the mass of a certain number of particles.	True, 72% (*N*=18%)	96 % (*N*=24)

More details emerged from a further descriptive analysis of the students' responses. While initially, I have assumed that knowing the percent of correct/incorrect answers would justify the intervention well, I decided to zoom more in-depth into how students solved the problem and what the errors were. Out of the 20 students who correctly converted 2 kg of sodium to the number to moles, (20%, *N*=4) applied dimensional analysis and 80% (*N*=16) constructed proportions. This results support the purpose of the study and the undertaken theoretical framework. It is to note that the students were free to use any method of their choice to conduct the conversions. After comparing with pretest results, the researchers noted that two of these students who used the dimensional analysis during the pre-test used proportions on the post-test. Zooming even farther it was also curious to learn what caused the remaining five students not succeed on the conversion problem. Two of the students did not attempt to solve it, two included Avogadro's number in the dimensional analysis and got lost during the conversion process, and one student incorrectly converted kilograms to grams which resulted in incorrect mole computations. A strong association of the unit of the mole to Avogadro's number still prevailed in these students' minds despite not highlighting this constant during the instructional unit. Misuse of the Avogadro's number was also mentioned by other researchers (e.g. Furio et al., 2002). The percentages of students who correctly interpreted the unit of the mole (*N*=24, 96%), molar mass (86%, *N*=22) and atomic mass (90%, 23) was also higher on the posttest. It is believed that bringing forth students' algebraic reasoning in a manner consistent with what they study in mathematics can be helpful in quantifying chemistry concepts. While the same size of the group under investigation was small, it is believed that the encouraging results can be still used by curriculum policy makers and other stakeholders to design science teaching materials that students will understand and enjoy studying.

Summary

The purpose of this study was to find out if using proportions and fundamental units can help students understand the conversions and the virtue of the idea of substance amount expressed in the units of moles. While the focus of the study was to develop conversion techniques, understanding the idea of molar mass and atomic mass was also exploited. I hypothesize that augmenting the conceptual definitions of molar and atomic mass helped students understand the meaning of the units and set up algorithms for conversions. This study also supports prior recommendations (see Niaz, 1987; Tullberg et al., 1994, Soon's et al. 2011), Scott's, 2012) that a consistent integration of algebraic operations and scientific terminology during conversions helps students learn these processes. While the prior research suggested such integration, this study attempted to provide concrete approaches that proved to be promising. It might be misleading to assume that students will figure out by themselves how to integrate the rules of algebra in the quantification processes. The integration phases must be developed carefully and delivered with attention to all nuances; algebraic and scientific (Sokolowski, 2018).

In sum, it can be suggested that providing students with methods whose algebraic structures are familiar to them can serve as a way of improving not only the mole understanding but also other computations, not necessarily related to conversions. I realized that perhaps there could be more pre-test/post-test items assessing the intervention from a wider angle. However, it is believed that the quantitative/qualitative nature of the items provided enough evidence to access the intervention. I hope that this study will initiate further research, perhaps with chemistry students to assess students' progress on other areas requiring proportions and these conversion phases that did not surface in this study. Furthermore, unifying the methods of teaching the mole across chemistry and physics, would deepen not only understanding of the mole but also improve students' general STEM disposition.

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