EJPCE Eurasian Journal of Physics and Chemistry Education

Eurasian J. Phys. & Chem. Educ. 5(2): 111-119, 2013

journal homepage: http://www.eurasianjournals.com/index.php/ejpce

Effect of Poinsot Construction in Online Stereo 3D Rigid Body Simulation on the Performance of Students in Mathematics and Physics

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Received: 09 March 2013- Revised: 30 April 2013- Accepted: 10 May 2013

Abstract

The current paper aims at presenting the effects of free online stereoscopic 3D simulation developed by the author on the performance of students in mathematics and physics. The simulation visualizes the Poinsot construction in free rigid body motion. The student is assisted in understanding the famous construction and in better comprehending the Newtonian mechanics and mastering its underlying mathematical model. The Poinsot construction is rendered in stereo 3D graphics in the web browser and the simulation shows the construction's inherent elements, such as invariant ellipsoids, invariant plane, polhode, herpolhode, etc. The latter are watched along with a large number of involved parameters: vectors and scalars. The presented material is directed towards university students taking the Analytical (Mathematical) Mechanics courses in the Faculty of Mathematics and Informatics and students taking the Theoretical Physics and General Physics courses in the Faculty of Physics in Sofia University, Bulgaria, but is not limited to use in other universities due to simulation's free unrestricted access on the Internet. The software was tested and its effectiveness was ascertained through experimental and control groups. Data collected in such experiments is presented in order to support the relevance of the study. Stereoscopic 3D simulations are a fruitful method for observation of phenomena hard to realize in laboratory conditions such as weightlessness. The simulation, discussed in this paper can be viewed and used from http://ialms.net/sim/ web address.

Keywords: Poinsot construction, Simulation of free rigid body motion, Mechanics in stereo 3D e-learning.

Introduction

This article intends to present a free-access online stereoscopic 3D simulation involving the celebrated Poinsot construction. The simulation software was developed by the author of this paper and has free and non-limited access to everyone and anyone on the Internet. The latter construction was introduced more than one hundred years ago by the French mathematician and physicist Louis Poinsot (Poinsot, 1834). Since then it helps students visualize and understand the complex matter of free rigid body motion phenomenon.

The simulation demonstrates the Poinsot construction, thus providing an ample foundation for teaching and learning this construction, understanding the Newtonian mechanics, mastering its underlying mathematical formalism and also assuring a method for observation of phenomena hard to realize in laboratory conditions – e.g. the absence of gravity. The Poinsot construction is represented in an interactive stereo 3D graphics simulation and its inherent elements, such as invariant ellipsoids, invariant plane, polhode, herpolhode, etc., are shown along with a large number of vectors, scalars and trajectories. User manual of the simulation is available at http://ialms.net/sim/3d-rigid-body-simulation-tutorial

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ISSN: 1306-3049,

Poinsot construction physical insight provided by the interactive stereo 3D visualization

The mathematical formalism of rigid body motion and Poinsot construction is presented in a number of books and papers such as José and Saletan (1998), Hand and Finch (2002), Landau and Lifshitz (1976), Goldstein, Pool and Safko (2001), Arnold (1989) and Whittaker (1937). The author's intention was to bring Poinsot construction in motion visualized using 3D graphics. Thus the formalism may be better comprehended with the assistance of the simulation. This construction was a helpful and valuable tool in teaching rigid body motion for decades. Its value has been since proven many times and hence it is taught in theoretical mechanics and analytical mechanics courses in universities throughout the world and is presented in a number of books among which Symon (1971), Taylor (2005), Kibble and Berkshire (2004), Van Name (1958) and Lurie (2002).

Why is this construction valuable? Obviously, because the student may "feel" the constants that govern free rigid body motion. The physical insight is far beyond demonstrating the mathematical formulas graphically. It is the visual perception of a real phenomenon or as Louis Poinsot said in his book "*Théorie Nouvelle de la Rotation des Corps*" (Poinsot, 1834):

> *"Everyone makes for himself a clear idea of the motion of a point, that is to say, of the motion of a corpuscle which one supposes to be infinitely small, and which one reduces by thought in some way to a mathematical point."*

The clear notion created by visual perception of the phenomenon leads the student over and beyond the mathematical formalism. It leads them directly to the phenomenon's real behavior. The student may derive conclusions of the motion not through recreating the motion as a mathematical consequence of its formal description, but in reverse – observe how the body actually moves and derive conclusions and formalize its behavior mathematically as disclosed in Zabunov (2010). This is an inverse approach of teaching rigid body motion. It follows the correct cognitive path of the perception of physical phenomena – from the phenomenon observation to formalism. Furthermore, this approach leads to better understanding of the underlying mathematical apparatus by students.

Finally, it should be noted that no simulation may substitute reality, but it may resemble reality to a given extent needed for proper education.

Motivation

The author was motivated in his work as a consequence of conducting an exhaustive survey and research in the field of interactive 3D physics simulations. The author realized that applications of e-learning systems in mechanics using stereoscopic 3D real-time approach almost did not exist (see 'Web sites' at the end of article). On the other hand, Poinsot construction has been hardly anywhere presented in e-learning physics simulations part of the educational avenue. Another strong requirement to the developed simulation was to conform to the modern e-learning principles and demands, such as platform independence (hardware and software), no download or special installation steps needed, distant access in the Internet environment through viewing the simulation in a web browser, etc. All these requirements have been fulfilled by this e-learning tool. The simulation's only software requirement is an installed Adobe® Flash® Player in the used web browser. The Flash Player is free and widely present on personal computer systems and would hardly ever demand from the user to perform this install as a prerequisite of simulation utilization. As a supplement, the feature of stereoscopic vision realized with very low priced equipment should be added. The needed prerequisites are a pair of standard red-cyan anaglyph glasses that cost around $1 only and no special hardware needed of any kind.

Another point of strong motivation was the recent interest in simulations and analysis of spacecraft stability. There are a number of publications on that matter, for example Coron and Kerai (1996), Kerai (1995) and Morin, Samson, Pomet and Jiang (1995).

The design of adaptive control of spacecraft dynamics has led to great success in the development of such controllers. Furthermore, there is another problem, namely the observer problem, which is much more difficult. It is the task of estimation and parameter identification followed by control. A rigid body simulation along with Poinsot construction interactive visualization is a fruitful basis for learning rigid body motion and experimenting by the students in the spacecraft control matter.

Comparison to other modern physics e-learning simulations

A few examples are drawn. There are more mechanics simulations than the ones described below, but those are no principally different and do not offer much further variety of features (see 'Web sites' section). The examples follow:

1. Examples of valuable 3D simulations are the *Massachusetts Institute of Technology* (MIT) 3D physics simulations. These simulations do not deal with the free rigid body motion, but nevertheless are a very good example of beneficial to students physics 3D simulations. MIT simulations offer Internet access and 3D graphics in a web browser. The simulations are interactive and can be controlled to certain extent by the user. The 3D scene may be observed from different angles. The motion of the molecules is clear and pleasant to observe.

2. Another example is the software product ThreeDimSim, which has free test version. This software platform is a desktop application and it offers simulation of mechanical systems of rigid bodies (mechanisms). The user may setup the initial conditions for various mechanical settings. This is achieved by setting rigid bodies' dimensions and joints between them. The environment is flexible and allows the user to observe the behavior of complex mechanical systems.

3. Yet another example is the software product NewtonPlayGround v1.53 (free test version). It performs similarly to ThreeDimSim letting the user setup mechanical systems of rigid bodies with different shapes, a number of variants for joints and external forces acting. The interface is easy to use and understand.

While these tools are disclosing a very substantial interest among the educational community in 3D mechanics and physics simulations, they present a front of development that demands new approaches and new scenarios to be realized. The presented simulation in the current paper moves a few steps further by representing vectors as 3D arrows and prints the values of numerous parameters describing the simulated process. Free rigid body motion is inherently conservative and the accumulation of numerical integration errors is not counteracted through friction. It has to be canalized in the proper manner not to violate the studied physical laws. Due to this requirement the free rigid body simulation is stabilized along two invariants – kinetic energy of rotation and angular momentum. Furthermore, a stereoscopic view mode is offered to enhance the perception of the 3D-scene by the observer and to help understand the spatial relations and orientation of the viewed elements. Finally, to enable an in-depth analysis and understanding of the free rigid body motion, the simulation shows the Poinsot construction with all of its elements.

Interactivity as means of teaching the Poinsot construction

Perhaps the most valuable feature of the presented simulation, along with the stereoscopic view mode, is its interactivity. Students may influence the body motion by setting different initial conditions and thus observe the Poinsot construction in certain cases determined by their needs for investigation and their imagination. This approach is obviously close to the modern concept of constructivism in education and tries to develop it in the avenue of interactive 3D visualization of phenomena studied in the university physics courses.

The Poinsot construction

The current presentation will not engage into mathematical formulations of rigid body motion with the pure intention not to make the text cumbersome and crammed with formulas. The reader may attain appropriate acquaintance with the mathematical formalization of the discussed problems in the literature such as Poinsot (1834), José et al. (1998), Hand et al. (2002) and Landau et al. (1976).

In free rigid body motion angular momentum vector $\vec{L}$ is constant by value and magnitude. The second invariant in free rigid body motion is the kinetic energy of rotational motion E_{Krot}. Both invariants lead to constants of motion known as constraint ellipsoids ε_ω and ε_L and invariable plane γ (Arnold, 1989). The constraint ellipsoids are named after the two vectors: angular momentum $\vec{L}$ and angular velocity ε_ω. Vector $\vec{L}$ is constraint to point on ε_L and vector $\vec{\omega}$ is constraint to point on ε_ω. These constraints are direct consequence of the two constants of motion mentioned above. The Poinsot constraint ellipsoid ε_ω and invariable plane γ are shown on Figure 1.

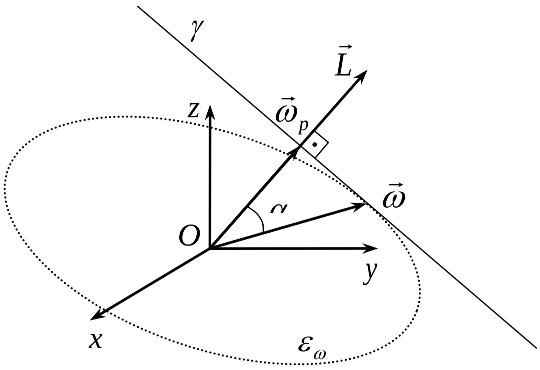


Figure 1. Poinsot construction in the space reference frame

The constraint ellipsoid is connected to the rigid body, i.e. it is static in the body reference frame and its orientation and shape depend on the moment of inertia tensor and the kinetic energy of rotation 1, 6]. A given moment of inertia tensor (certain homogenous body shape and mass) and the current kinetic energy of rotation (constant) define the ellipsoid fully. The angular momentum vector $\vec{L}$'s direction is always coincident with the direction of the invariable plane γ normal vector i.e. $\vec{L}$ is equal to the γ's normal vector multiplied by a non-negative scalar. The angular velocity vector $\vec{\omega}$ is constraint to point on ε_ω, as mentioned above, but it is also constrained to point on the invariable plane γ (Figure 1). The trajectory of the angular velocity vector $\vec{\omega}$ on the invariable plane is a curve called herpolhode and the trajectory of the angular velocity vector $\vec{\omega}$ on the constrained ellipsoid ε_ω is a curve called polhode (Poinsot, 1834 and Arnold, 1989). On Figure 2 are presented the herpolhode and space coordinate axes in black shade and the angular velocity vector in light grey shade.

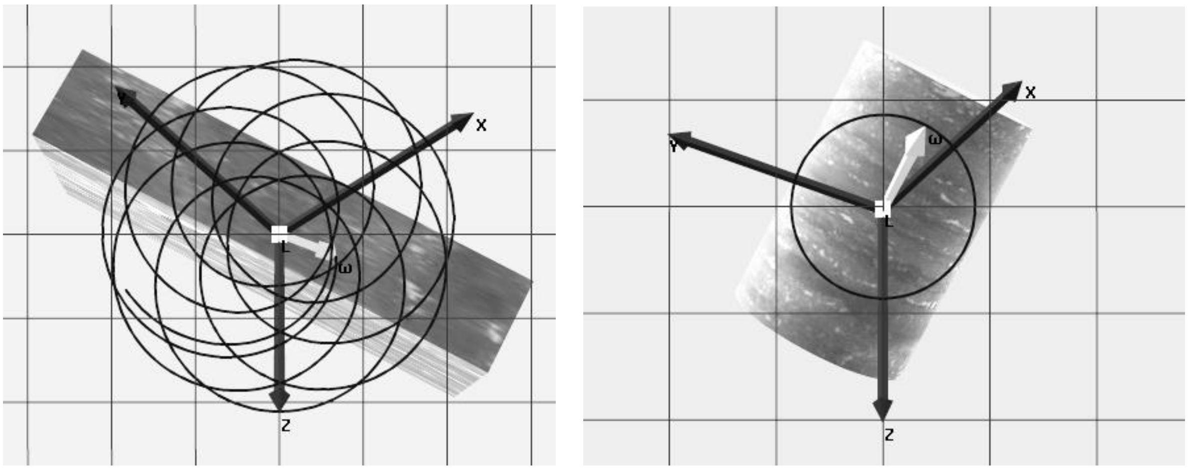


Figure 2. Poinsot construction showing only the invariable plane γ and the herpolhode. On the left: general case (orthogonal parallelepiped); on the right: cylinder (symmetric top). Color online.

As mentioned above, the herpolhode is the curve drawn by the angular velocity vector $\vec{\omega}$'s tip over the invariable plane γ. The Poinsot construction shows that vector $\vec{\omega}$ would always point on the invariable plane γ and on the constraint ellipsoid ε_{ω} surfaces at the same time (Figure 2. and Figure 3.). At that point both surfaces have coincident normal vectors, i.e. the ellipsoid intersects the plane tangentially and rolls over the plane. It is also visible that this rolling happens without slippage because the angular velocity at the point of intersection is zero.

The simulation allows the visualization of the invariable plane and the herpolhode separately from other Poinsot construction elements. The herpolhode is a plane curve with cycloidal form and degrades into a circle under certain initial conditions, i.e. when the rigid body is a symmetric top. The herpolhode may or may not be a closed curve after a finite number of rotations. The herpolhode is also restrained between two concentric circles with their centers lying on the direction line of angular momentum vector.

The simulation is further capable of demonstrating the constraint ellipsoid and the polhode separately from other Poinsot construction elements. The polhode is a curve drawn by the angular velocity vector $\vec{\omega}$'s tip over the constraint ellipsoid. Figure 3 shows the polhode and the space coordinate axes in dark shade. The angular velocity vector is rendered in light grey shade. The angular momentum vector is shown in black shade on the left and in white shade on the right of Figure 3. The polhode is a closed taco-shaped curve that degrades to circle under certain initial conditions (symmetric top).

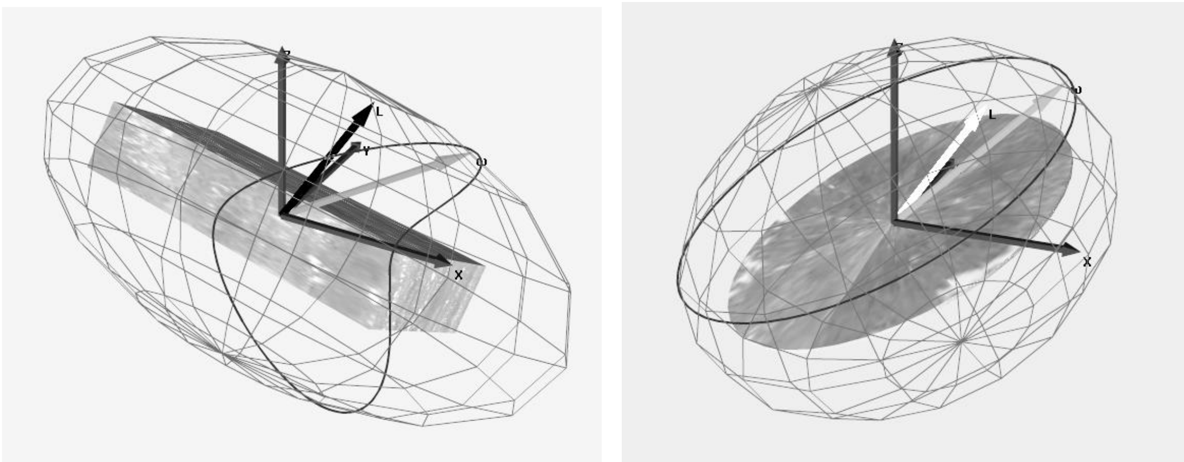


Figure 3. Poinsot construction showing only the constraint ellipsoid and the polhode. On the left: general case (orthogonal parallelepiped); on the right: disc (symmetric top). Color online.

Statistical Review

A number of statistical studies were conducted with the goal of examining the effectiveness of the simulation with students taking the Mathematics and Physics courses at Sofia University. An extract of such a study is presented through graphical charts in Table 1. The students' responses mentioned in Table 1 were summoned after a simulation presentation being a method of blended teaching in lectures of Physics at Sofia University. The anticipated simulation effects were in two groups: the first group was related to forming of concepts such as vector quantities (force, angular momentum, acceleration, trajectories of motion, etc.). The utilized stereo anaglyph 3D glasses (see Figure 4.) additionally helped in improving the learning of concepts and the relations between them. The second group of effects comprised the inertial properties of rigid bodies. Different types of bodies and their free motion was demonstrated along with the Poinsot construction and its inherent elements. The free precession phenomenon was studied and analyzed. The students' notion of free rigid body motion and its dependency on initial conditions was broadened. Further, the students were encouraged to attain self-dependant study using the simulation that was accessible free without limitations from the Internet at web address http://ialms.net/sim. As a result of this self-conduced learning, responses and comments were received from student recommending improvements and refinements.

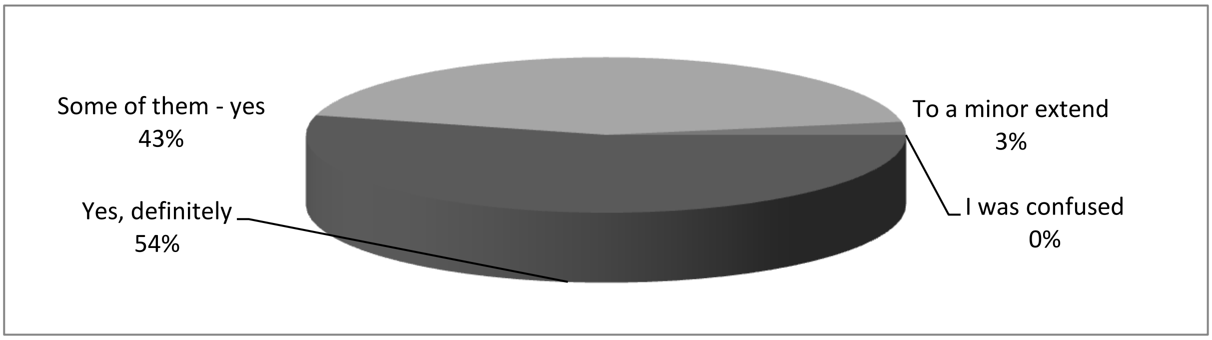
Extract from results of questionnaires answered by students in two of the many occasions during Physics lectures is presented in Table 1 (38 students were participating).

The answers speak for themselves. Nevertheless, systematic statistical analyses were conducted asserting the effectiveness of the simulation. Explicit statistical proofs will not be exposed in order not to overburden the presentation with formulas and numbers.

Figure 4. Students observing the Poinsot construction in simulation using stereo anaglyph 3D glasses in lectures at Sofia University.

Table 1. An extract of students' responses to questions regarding the effects of studying the Poinsot construction with the help of the 3D stereo simulation.

Did the simulations help you in clarifying the notion of quantities and laws in mechanics?



Would you like to continue using such simulations in the teaching process?

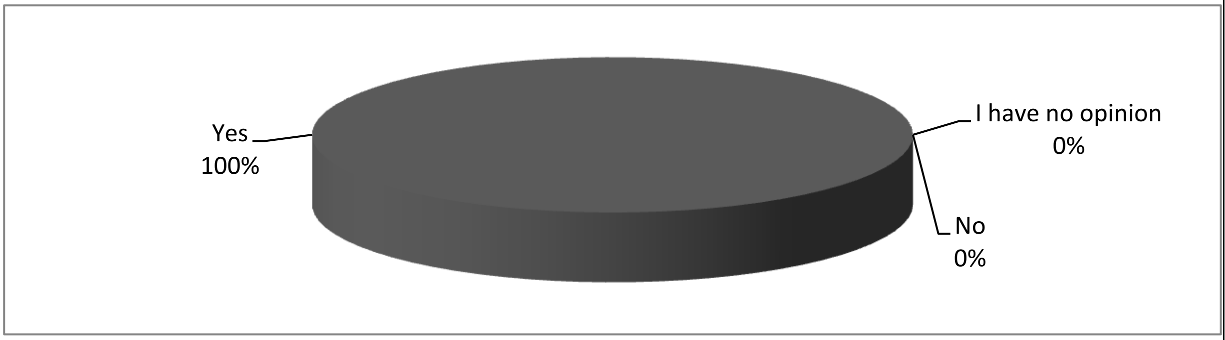
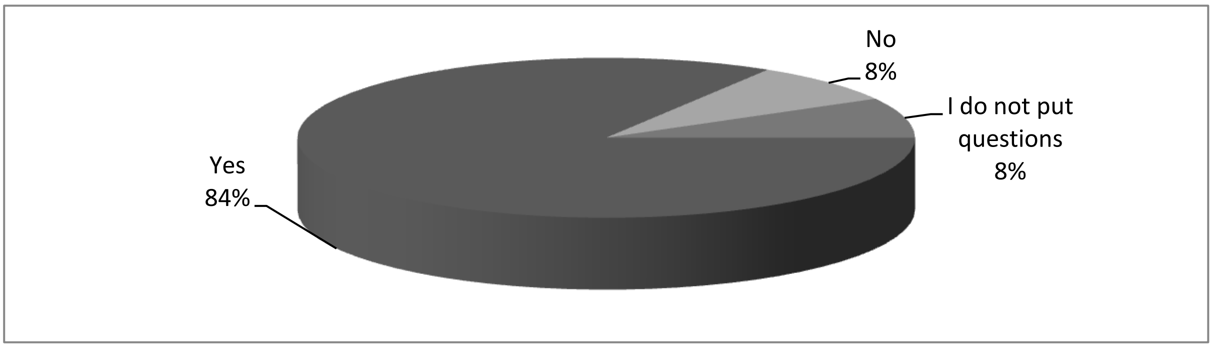
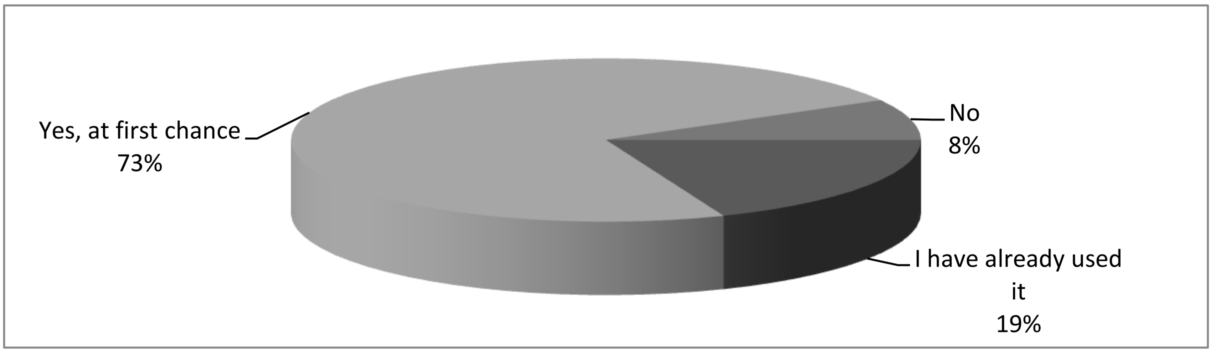
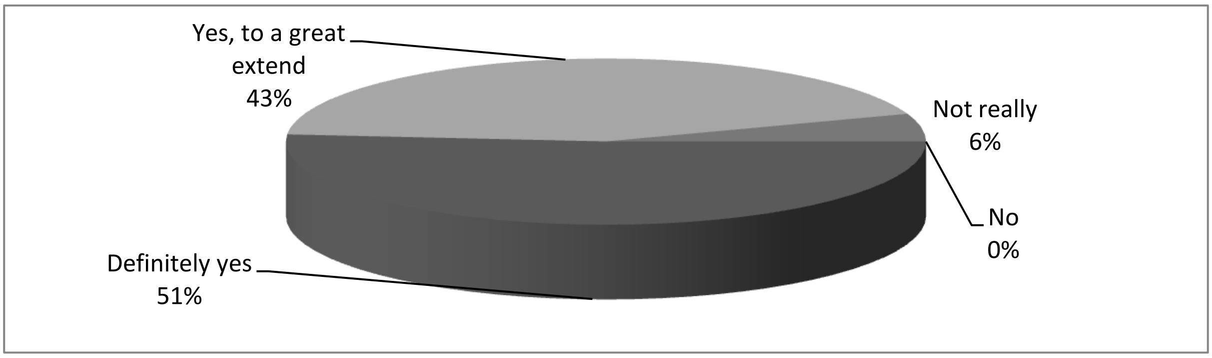


Table 1. Continue

Did the simulation respond to questions that arose during lectures? 
Would you use the simulation in self-conducted learning? 
Do you consider that the implementation of such simulations in teaching Physics creates positive motivation for learning? 

From the reported statistical observations it can be concluded that:

1. The implementation of simulations promotes the motivation and intensifies the interest towards the tutored material.

2. The observation and the interactive and self-conducted work with the simulation make better the comprehension of concepts and laws of the rigid body mechanics.

Conclusion

The Poinsot construction is a fruitful approach to learning and understanding the rigid body motion and also the basic principles in the classical mechanics taught in the theoretical and analytical (mathematical) mechanics courses. The drawback is that these constructions are hard to visualize in imagination and they are difficult to understand without visualizing. This gap is successfully filled using the free online stereo 3D simulation and the student may observe the Poinsot construction in motion with all accompanying vectors and reference frames.

As noted earlier, although a simulation could never duplicate reality, the impossibility to realize the studied process and observe its inherent elements in laboratory conditions represents another reason why such a simulation gives good results while used for teaching students in the universities.

Still another benefit is the student's attitude towards the problem. It becomes individualized and lets him or her place the simulation in states, which are most relevant in demonstrating the sought knowledge.

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